PRESSURE VESSELS

Pressure vessels can either be cylindrical or spherical (hydraulic cylinders, pipes, boilers and water or air tanks) are used in the industry to hold liquids or gasses under pressure. High internal and external pressures generate both radial and tangential stresses which vary throughout the radius of the cylinder and are essential to consider when analyzing the structural integrity of such vessels.

Pressure vessels are containers designed to hold gases or liquids at a pressure significantly different from the ambient pressure (greater than 0.7 atm).

Design Considerations

- Stress Concentration:
  - High stress regions near openings and welds require careful analysis.
- Environmental Factors:
  - Creep, fatigue, and corrosion must be accounted for.
- Codes and Standards:
  - Designs must comply with industry standards (e.g., ASME, ISO).

Proper analysis of these stresses is crucial to ensuring the structural integrity and safety of pressure vessels, which are classified as either thin-walled or thick-walled based on the ratio of their inside diameter to wall thickness.

TYPES:

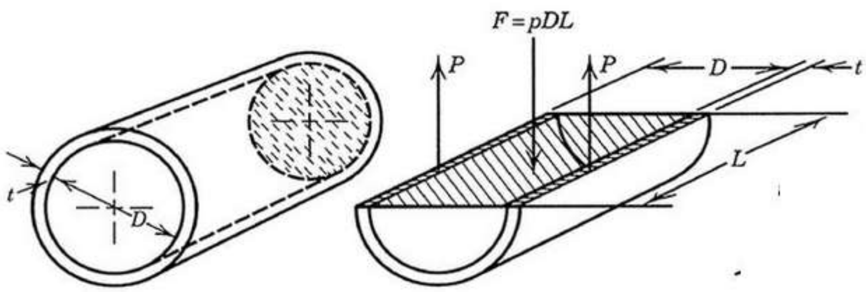
A. Thin-walled pressure vessel

A pressure vessel is considered to be thin-walled when the ratio of the inside diameter, D_i to the wall thickness, t is equal or greater than 10, or that is $\frac{D_i}{t} \geq 10$.

For cylindrical thin-walled pressure vessel:

For tangential or hoop stress, S_t:

To analyze hoop stress, we consider a **free-body diagram** of half of the cylinder. The internal pressure PPP exerts an outward force over the **cross-sectional area**.



Where: L = Length
D = Internal Diameter
t = Wall thickness
p = Fluid pressure inside the vessel

Summation of Forces for equilibrium: F = 2P

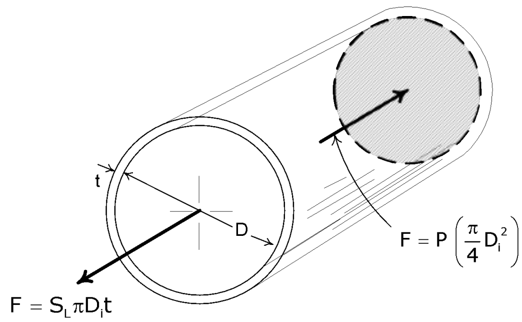
The force due to internal pressure: F = pA = pDL ; $A = DL$

Resisting force in the material: $P = S_t A = S_t(tL)$; $A = tL$

For equilibrium: $pDL = S_t(tL)$

Hence, the tangential or hoop stress is: $S_t = \frac{pD}{2t}$

For longitudinal or axial stress, S_L:



Summation of Forces for equilibrium: F = P

The force due to the internal pressure is: F = $pA = p\left(\frac{\pi}{4}D^2\right)$; A = $\frac{\pi}{4}D^2$

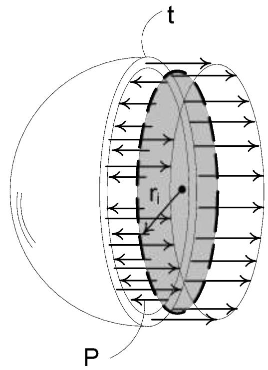
Resisting force in the material: $P = S_L A = S_L(\pi D t)$; $A = \pi D t$

For equilibrium: $p\left(\frac{\pi}{4}D^2\right) = S_L(\pi D t)$

Hence, The longitudinal stress is: $S_L = \frac{pD}{4t}$

If the tank is over-pressurized or experiences material fatigue, it will likely develop cracks along the length because hoop stress is the dominant stress and is twice as high as the longitudinal stress , leading to a horizontal rupture.

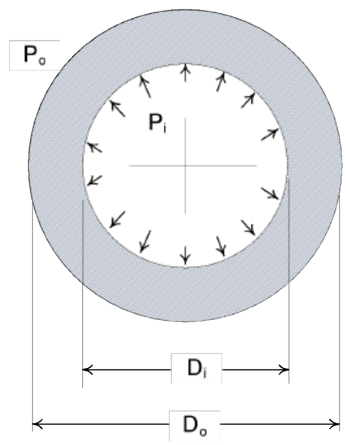
For spherical thin-walled pressure vessel:



$$S_t = S_L = \frac{pD}{4t}$$

B. Thick-walled pressure vessel

A pressure vessel is considered to be thick-walled when the ratio of the inside diameter, D_i to the wall thickness, t is less than 10, or that is $\frac{D_i}{t} < 10$.



Lame's Formula:

Lame's formula is used to calculate the stresses in thick-walled pressure vessels that are subjected to both internal and external pressures. Thick-walled vessels are used when the wall thickness is large compared to the vessel's radius, and this causes the stresses to vary through the wall. A set of equations developed by French mathematician Gabriel Lamé.

Maximum tangential stress (hoop stress) at the inside: $$S_{ti} = \frac{P_i\left(r_o^2 + r_i^2\right) - 2P_o r_o^2}{r_o^2 - r_i^2}$$

Maximum tangential stress at the outside: $$S_{to} = \frac{2P_i r_i^2 - P_o\left(r_o^2 + r_i^2\right)}{r_o^2 - r_i^2}$$

Maximum shearing stress at the inside surface of the cylinder: $S_{Si} = \frac{r_o^2(P_i + P_o)}{r_o^2 - r_i^2}$

Maximum longitudinal stress (Closed ends): $S_L = \frac{P_i r_i^2 - P_o r_o^2}{r_o^2 - r_i^2}$

Maximum wall thickness: $t = \frac{D_i}{2}\left[\sqrt{\frac{S_t + P_i}{S_t - P_i}} - 1\right]$

Where:
P_i = internal pressure
P_o = outside pressure
r_i = inside radius
r_o = outside radius
t = wall thickness
D_i = internal diameter
D_o = outside diameter
L = length of the pressure vessel

Thick-walled pressure vessels are used in various applications, some of these are:

High-Pressure Gas Cylinders	Oxygen tanks for hospitals and welding applications Compressed natural gas (CNG) storage tanks in vehicles Hydrogen storage tanks for fuel cell vehicles SCUBA diving tanks (used underwater)
Hydraulic Cylinders	Hydraulic jacks (car jacks, construction equipment) Hydraulic press cylinders (metal forming, industrial manufacturing) Aircraft landing gear cylinders
Deep-Sea Submarine Hulls	Manned submarines (military and research submarines) Deep-sea diving vessels (e.g., Alvin, Trieste, DSV Limiting Factor) ROVs (Remotely Operated Vehicles)
Nuclear Reactor Pressure Vessels	Nuclear fission reactor vessels (used in power plants) Fusion reactor containment chambers (experimental reactors)
High-Pressure Steam Boilers	Steam boilers in power plants Industrial steam generators (chemical processing, refineries)
Automotive Parts	Hydraulic Cylinders in Power Steering and Suspension Systems Brake Master Cylinder and ABS Hydraulic Modulator Turbocharger and Supercharger Casings

Classification of Pressure Vessels

Pressure vessels are classified based on the following criteria:

1. Thickness of Vessels

- Thin-Walled Vessels: The wall thickness is small compared to the vessel's diameter. Hoop stress is assumed constant across the thickness.
- Thick-Walled Vessels: Hoop stress varies across the wall, being maximum at the inner surface.

2. End Construction

- Open-Ended Cylinders: Subject to hoop and radial stresses only.
- Closed-Ended Cylinders: Additional longitudinal stress acts due to the closed ends.
  - Restraint at the ends induces additional stresses, increasing design complexity.

3. Material of Construction

- Materials can be brittle (cast iron) or ductile (steel).
- Selection depends on the operational conditions and safety considerations.

4. Geometrical Shape

- Common shapes:
  - Cylindrical
  - Spherical
  - Conical
  - Ellipsoidal
- Thin-walled designs are generally preferred for simplicity, but spherical vessels are ideal for uniform stress distribution.

5. End Configuration

Hemispherical Ends:

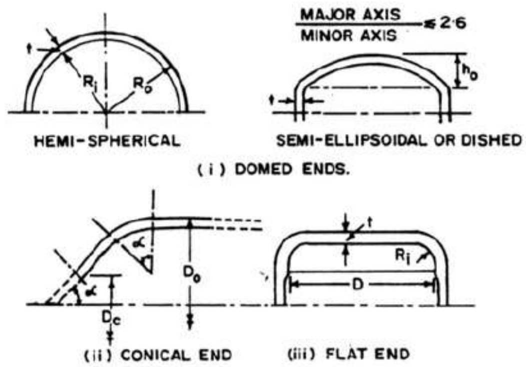
- Best for minimizing stress.
- Expensive to fabricate.

Ellipsoidal Ends:

- Slightly less effective but more economical.

Flat Ends:

- Inexpensive but prone to high stress.



6. Method of Fabrication

- Fabrication methods:
  - Welding (common for high-pressure applications)
  - Riveting (used when frequent maintenance is required)
  - Bolting
  - Seamless construction for tubes and pipes.

7. Service Conditions

- Factors affecting design:
  - Temperature (creep occurs above 350°C)
  - Corrosive environments
  - Pressure fluctuations
  - Appropriate material selection and stress analysis are critical.

8. Fired vs. Unfired Vessels

- Fired vessels are exposed to combustion (e.g., boilers).
- Unfired vessels do not involve combustion (e.g., storage tanks).

WIRE ROPES

Wire ropes are fundamental components in various mechanical, industrial, and construction applications. At their core, wire ropes are constructed by twisting multiple wires into strands, and then helically laying these strands to form a rope. These wires may be metallic or non-metallic, and the configuration—including the number of strands, number of wires per strand, and the direction of lay—directly influences the rope's strength, flexibility, and resistance to wear and fatigue.

Wire ropes are made of flexible, twisted strands of wire that are usually laid over a core. They are used in elevators, cranes, as suspension cables, guy wires and hoisting. They are designed to combine strength, flexibility, and durability.

Current applications include:

Elevators
Mine hoists
Cranes
Oil-well drilling
Aerial conveyors
Tramways
Haulage systems
Suspension bridges

Construction:

- Made from **cold-drawn wires**.
- **Wires** are twisted into **strands**.
- **Strands** are helically wrapped around a **core**, which can be:
    - **Hemp or pulp** (for flexibility).
      IWRC (Independent Wire Rope Core) for added strength and crush resistance.

Lay of Wire Ropes

Wire rope "lays" are descriptions of how wires and strands of the rope are laid in a helix. Regular lays and Lang lays are two ways of laying the wire within each strands

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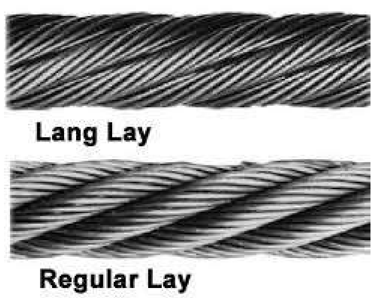
<td>
</td>
<td>Lang Lay: Strands and wires twisted in the same direction. More flexible, wears better on drums and sheaves.

Regular Lay: Strands twisted in one direction, wires in the opposite. Offers better resistance to rotation.</td>
</tr>
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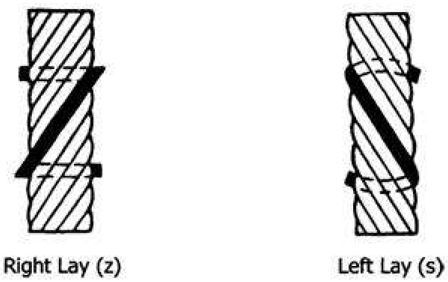
Lay Direction

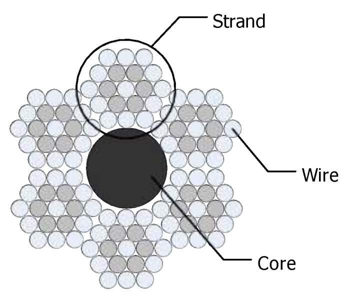
An essential aspect of wire rope design is the lay, which refers to the direction and manner in which wires and strands are helically wound. The lay may be right-hand or left-hand, and the configuration can be either regular lay—where the wires are laid in the opposite direction to the strands—or Lang lay—where both wires and strands are laid in the same direction. Regular lay ropes offer better resistance to crushing and easier handling, making them suitable for applications involving frequent bending. Lang lay ropes provide increased flexibility and wear resistance, especially when the rope runs over drums or sheaves. However, they are more prone to untwisting and must be handled carefully.

Right Lay (Z) and Left Lay (S) in wire rope is determined by the direction the strands spiral around the core, and you can visually identify it using the pattern of the helix



Wire Rope Designation

A 6 x 19 RH wire rope has 6 strands each having 19 wires and right hand lay. Rope size, Dr, is the **diameter** of the circumscribed circle around the rope.



The core of a wire rope plays a critical role in supporting the strands and absorbing stresses during loading. There are several core types: fiber cores (made from materials such as manila, cotton, or synthetic fibers), wire strand cores (WSC), and independent wire rope cores (IWRC). Fiber cores provide greater flexibility and cushioning but lower strength. WSC and IWRC, on the other hand, offer greater resistance to crushing and distortion. For high-load applications, IWRC is typically preferred. Notably, ropes with metallic cores are rated about 7½% stronger than those with non-metallic cores.

Wire Rope Materials:

Materials used in wire ropes are selected based on strength requirements and operating conditions. In increasing order of strength, common materials include iron, phosphor bronze, traction steel, plow steel, improved plow steel, and bridge rope steel. Traction wire rope, for instance, is specifically designed for railways and freight applications, while improved plow steel offers high abrasion resistance and is suitable for harsh service conditions. Galvanized ropes, while corrosion resistant, are about 10–15% weaker than bright (ungalvanized) ropes, although they are often preferred for outdoor or marine use.

- Iron (without 0.1% carbon)
- Traction
- Mild plow steel (MPS)

- Plow steel (PS)
- Improved plow steel (IPS)
- Very high strength (VHS)

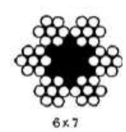
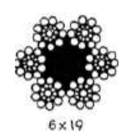
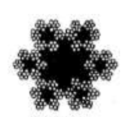
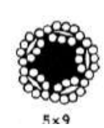
Ultimate Strength of Steel Wire Types

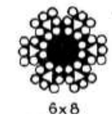
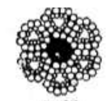
Material	Ultimate Strength (psi)
Iron	85,000
Cast Steel	170,000
Extra-strong Cast Steel	190,000
Plow Steel	210,000
Alloy Steel	230,000

Smaller wire sizes: 10–20% higher strength.
Due to curvature in manufacturing, final strength is 5–20% lower than the raw material.

Wire rope construction

Wire ropes are composed of multiple strands of wires, twisted in specific configurations to meet desired mechanical properties like strength, flexibility, wear resistance, and fatigue life.

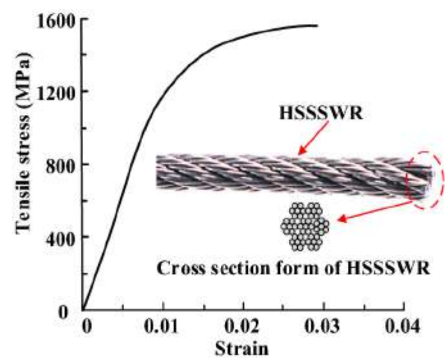
Construction Type	Description	Flexibility Category	
6 x 7	Coarse lay	Low flexibility, high wear resistance	
6 x 19	Seale construction	Medium flexibility	
6 x 37	Warrington construction	High flexibility	
6 x 6 x 7	Tiller rope	Very high flexibility	
5 × 9		Low to medium flexibility	

6 × 8	Flattened strand	Medium flexibility	
5 × 28		High flexibility, fine wires	
6 × 25		High flexibility, dense structure	

WIRE ROPES FUNDAMENTALS

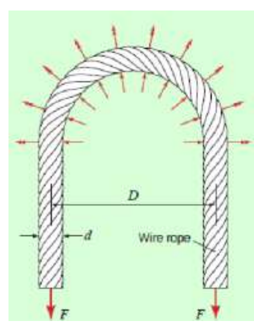
Extension of the wire rope under load (Hooke's Law), δ:

$$\delta = \frac{FL}{AE_r}$$



Geometric effect of curvature:

When a rope or belt bends around a drum/pulley of diameter D, the fibers near the outer surface must stretch more compared to those near the neutral axis. This stretch is proportional to the ratio of fiber distance from the neutral axis to the pulley diameter.



Hence;

$$\frac{\delta}{L} = \frac{d_w}{D}$$

Equivalent Bending Stress, S_b**:** $$S_b = \frac{E_r d_w}{D}$$

Equivalent Bending Load, F_b: $$F_b = \frac{A E_r d_w}{D}$$

Factor of Safety, F.S.: $$FS = \frac{F_U - F_b}{F_t}$$

Approximate weight of wire ropes, W: $W = 1.58d^2$

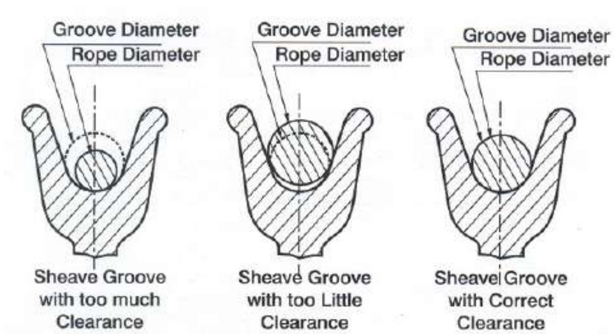
Approximate wire diameters and areas:

Rope	d_w	A
6 x 7	0.106 d	0.38 d^2
6 x 19	0.063 d	0.38 d^2
6 x 37	0.045 d	0.38 d^2
8 x 19	0.050 d	0.35 d^2

Where: d_W = wire diameter, in
d = diameter of the rope, in
A = total cross-sectional area of the rope, in^2
E_r = modulus of elasticity = 12 x 10^6 psi for steel ropes
D = sheave or drum diameter, in
L = length of the wire rope
F_u = breaking strength of rope
F_b = equivalent bending load due to curvature of the sheave or drum
F_t = tensile force due to the load (including acceleration of forces)
Refer to Table AT 28, Properties of Wire Rope, page 605: Design of Machine Elements 4th Edition by Virgil Faires

Sheave Grooves in Wire Rope Systems

The design of the sheave groove plays a critical role in the performance and longevity of wire ropes. If the groove radius is too large, the rope lacks support and flattens out, increasing fatigue. On the other hand, if the radius is too small, the rope becomes wedged and its rotation is restricted, concentrating wear along two lines. This uneven wear accelerates wire breakage and reduces rope life.

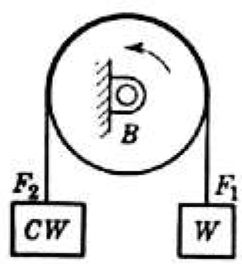


To avoid these issues, the groove should support the rope along nearly half of its circumference. The correct groove diameter is slightly larger than the rope's. The proper radius also depends on the rope diameter, with specific recommendations based on rope size.

Idlers and Guide Sheaves

Idlers and guide sheaves play an important role in supporting and directing ropes in mechanical systems. Although they are not responsible for power transmission like driving sheaves, their design significantly affects the performance and longevity of the rope. These components should be of generous proportions to minimize bending and reduce fatigue effects on the rope. Excessive bending can lead to flexural fatigue, which shortens the lifespan of the rope due to the repeated stress it experiences as it bends around the sheaves. While idlers and guide sheaves typically encounter less force compared to driving sheaves, allowing for somewhat smaller diameters, they must still be large enough to prevent excessive curvature. To ensure proper guidance, they should be grooved at the center to keep the rope aligned and designed with enough width to prevent the rope from slipping off sideways. This attention to design detail ensures smoother operation and enhances the durability and reliability of the entire rope system.

Traction Drives



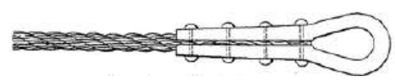
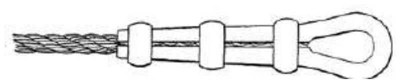
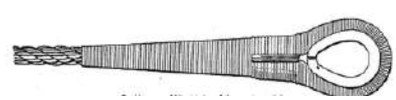
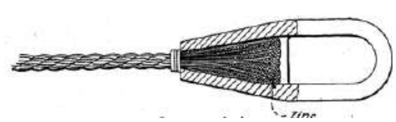
In traction drives, the friction between the sheave and the rope is utilized as the main driving force for transmitting motion. When the driving shaft rotates the sheave, the rope wrapped around it experiences two different tensions: the tight side tension F_1 and the slack side tension F_2, with $F_1 > F_2$.
The driving force that propels the load is the difference $F_1 - F_2$. For a system such as an elevator or hoist, W represents the load, while CW is the counterweight. The ratio of the two tensions is limited by the belt friction equation:

$$\frac{F_1}{F_2} = e^{f\theta}$$

where f is the coefficient of friction between the rope and the sheave, and θ is the angle of wrap in radians. This relationship shows that the greater the wrap angle or the higher the coefficient of friction, the larger the tension ratio that can be sustained without slipping. In practice, values of f vary depending on the materials and conditions of contact: for example, an iron or steel sheave with a dry rope may provide $f = 0.12$, while a rubber- or leather-lined sheave with a dry rope may provide up to $f = 0.495$. Thus, lining the sheave with rubber or leather significantly increases traction capability, allowing greater load transmission without slip. Refer to the paragraph in page 477: Design of Machine Elements 4th Edition by Virgil Faires

Wire Rope Fasteners

Wire rope fasteners are used to terminate or join ropes, but not all methods preserve the full strength of the rope. Spliced wire ropes should generally be avoided, as they only allow about 75% of the rope’s working strength compared to unspliced ropes. When terminations are required, various fastening methods are available, each offering different strength efficiencies.

Eye Shoe with Rivets	This fastener involves a metal eye fitting (shoe) that is attached to the end of the rope using multiple rivets. Strength and efficiency is Moderate — may not develop full rope strength due to possible slippage or localized stress at rivet points.	
Shoe with Driven Rings	Similar to the riveted eye shoe, but secured with metal rings that are driven onto the rope and shoe to clamp them together. Comparable to riveted shoes, possibly a bit more flexible in assembly but still doesn't reach 100% strength.	
Thimble and Clamps	**A thimble** (a grooved, tear-drop-shaped metal insert) is placed inside the rope loop to prevent crushing. **U-bolt clamps** are used to hold the rope in position. Approximately **75%–85%** of full rope strength.	
Splice with Thimble Seized	**rope splice** (weaving strands into each other) is formed around a thimble and secured in place with seizing wire or binding. Around **90%** of full rope strength if done properly.	
Wire-Rope Socket with Zinc	The rope’s strands are separated, cleaned, and placed into a socket filled with **molten zinc**, which solidifies and grips the wires securely. **100%** of rope strength when properly made.	

Properties and Strength

The principal properties of wire ropes include tensile strength, wear resistance, flexibility, and resistance to crushing and distortion. Strength is determined by the material and construction; preformed ropes and those with IWRC generally perform better under high load. Flexibility is enhanced by using smaller wires in larger numbers and adopting Lang lay. Resistance to crushing is improved by selecting ropes with IWRC or Seale constructions. Flattened strand ropes, by design, offer a broader surface contact area, reducing localized wear.

Tables from standards such as the U.S. Simplified Practice Recommendation 198-50 provide quantitative values for breaking strength (in tons), rope weight per foot, and comparisons among steel grades: mild steel, plow steel, and improved plow steel. Ropes with IWRC and improved plow steel consistently show higher performance ratings.

Safe Working Load and Factors of Safety

When designing or selecting wire ropes, safe working load and factor of safety (FS) must be considered. FS for standing rope applications typically ranges from 3 to 4, while for operating conditions (involving motion or lifting), it may vary from 5 to 12, depending on the hazard level. The rope's condition (new, worn, or corroded) also influences the selection. Engineers must also consider effects of friction, bending stress over sheaves, and the dynamics of acceleration/deceleration in hoisting operations.