

Answer Key

Shaft Design

1

$$P = \frac{2\pi T N}{60}$$

$$320 \text{ kW} = \frac{2\pi T (850 \text{ rpm})}{60}$$

$$T = 3.5950 \text{ kN-m}$$

Based on yield stress

$$S_y = \frac{16 T}{\pi D^3}$$

$$200000 \frac{\text{N}}{\text{m}^2} = \frac{16 (3.5950 \text{ kN-m})}{\pi D^3}$$

$$D = 0.0403 \text{ m}$$

Based on ultimate stress

$$S_u = \frac{16 T}{\pi D^3}$$

$$380000 \frac{\text{N}}{\text{m}^2} = \frac{16 (3.5950 \text{ kN-m})}{\pi D^3}$$

$$D = 0.0364 \text{ m}$$

Choose :

$$D = 40.3 \text{ mm}$$

For Short Starts

$$P = \frac{D^3 N}{38}$$

$$(120 \text{ kW}) \left(\frac{1 \text{ hp}}{0.746 \text{ kW}} \right) = \frac{\left[60 \text{ mm} \left(\frac{1 \text{ in}}{25.4 \text{ mm}} \right) \right]^3 N}{38}$$

$$N = 463.7388 \text{ rpm}$$

For Main SHAFT

$$P = \frac{D^3 N}{80} = \frac{(3.5 \text{ in})^3 (630 \text{ rpm})}{80}$$

$$P = 337.6406 \text{ hp}$$

For line shaft

$$P = \frac{D^3 N}{53.5}$$

$$130 \text{ hp} = \frac{(3 \text{ in})^3 N}{53.5}$$

$$N = 257.5926 \text{ rpm}$$

for the tangential speed

$$v = \pi D N = \pi (18 \text{ in}) \left(\frac{1 \text{ ft}}{12 \text{ in}} \right) \left(\frac{1 \text{ m}}{3.28 \text{ ft}} \right) \left(\frac{257.5926 \text{ rpm}}{60} \right)$$

$$v = 6.1681 \text{ m/s}$$

$$\theta = \frac{TL}{JG} \quad ; \quad \frac{\theta}{L} = \frac{T}{JG} \quad ; \quad J = \frac{\pi D^4}{32} = \frac{\pi (0.060 \text{ m})^4}{32}$$

$$J = 1.2723 \times 10^{-6} \text{ m}^4$$

Hence;

$$\frac{\theta}{L} = \frac{(1800 \text{ kN-mm}) \left(\frac{1 \text{ m}}{1000 \text{ mm}} \right)}{(1.2723 \times 10^{-6} \text{ m}^4) (80 \times 10^6 \text{ N/m}^2)}$$

$$\frac{\theta}{L} = \left(0.01768 \frac{\text{rad}}{\text{m}} \right) \left(\frac{180^\circ}{\pi \text{ rad}} \right)$$

$$\frac{\theta}{L} = 1.0132 \frac{\text{deg}}{\text{rad}}$$

$$hp = \frac{2\pi TN}{33000}$$

$$30hp = \frac{2\pi T (1300 \text{ rpm})}{33000}$$

$$T = (121.2026 \text{ ft-lb}) \left(\frac{12 \text{ in}}{1 \text{ ft}} \right)$$

$$T = 1454.4313 \text{ in-lb}$$

Solid shaft

Hollow shaft

$$\xi = \frac{16T}{\pi D_s^3}$$

;

$$\xi = \frac{16TD}{\pi(D^4 - d^4)}$$

equating;

$$\frac{16T}{\pi D_s^3} = \frac{16TD}{\pi(D^4 - d^4)}$$

$$\frac{1}{D_s^3} = \frac{D}{D^4 - d^4}$$

$$D_s^3 = \frac{D^4 - d^4}{D}$$

$$D_s^3 = \frac{(5 \text{ in})^4 - (3.5 \text{ in})^4}{5 \text{ in}}$$

$$D_s = 4.5627 \text{ in}$$

$$\theta = \frac{TL}{JG} \quad ; \quad J = \frac{\pi D^4}{32} = \frac{\pi (1/4 \text{ in})^4}{32} = 3.8349 \times 10^{-4} \text{ in}^4$$

subst;

$$\theta = \frac{(28 \text{ in} - 16)(10 \text{ in})}{(3.8349 \times 10^{-4} \text{ in}^4)(11000000 \text{ lb/in}^2)}$$

$$\theta = (0.06638 \text{ rad}) \left(\frac{180^\circ}{\pi \text{ rad}} \right)$$

$$\theta = 3.8030^\circ$$

$$\tau_s = \frac{16T}{\pi D^3}$$

$$50 \frac{\text{N}}{\text{mm}^2} = \frac{16 (100 \text{ N-m}) \left(\frac{1000 \text{ mm}}{1 \text{ m}} \right)}{\pi D^3}$$

$$D = 21.6770 \text{ mm}$$

$$L = 8.95 \sqrt[3]{D^2}$$

$$15 \text{ ft} = 8.95 \sqrt[3]{D^2}$$

$$D = 2.1697 \text{ in}$$

and;

$$D = 4.6 \sqrt[4]{\frac{P}{N}}$$

$$2.1697 \text{ in} = 4.6 \sqrt[4]{\frac{P}{280 \text{ rpm}}}$$

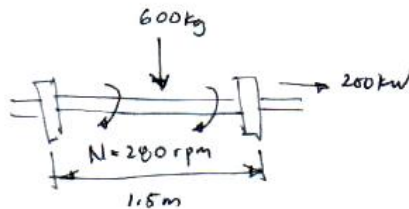
$$P = 13.8588 \text{ hp}$$

$$L = 8.95 \sqrt[3]{D^2}$$
$$8.95 = 8.95 \sqrt[3]{D^2}$$
$$D = 0.8451 \text{ in}$$

$$S_t = \frac{16}{\pi d^3} \left(M + \sqrt{M^2 + T^2} \right)$$
$$= \frac{16}{\pi (0,120 \text{ m})^3} \left(3 \text{ kN-m} + \sqrt{(3 \text{ kN-m})^2 + (8 \text{ kN-m})^2} \right)$$

$$S_t = 34023.8011 \text{ kPa}$$

$$S_t = 34.0238 \text{ MPa}$$



$$M = \frac{FL}{4} = \frac{(600 \text{ kg}) \left(\frac{9.81 \text{ m/s}^2}{\frac{1000 \text{ kg-m}}{\text{KN-s}^2}} \right) (1.5 \text{ m})}{4} = 2.2073 \text{ kN-m}$$

and;

$$P = 2\pi TN$$

$$200 \frac{\text{kN-m}}{\text{s}} = 2\pi T \left(\frac{280 \text{ rpm}}{60} \right)$$

$$T = 6.8209 \text{ kN-m}$$

Hence;

$$S_s = \frac{16}{\pi D^3} \sqrt{M^2 + T^2}$$

$$45000 \text{ kPa} = \frac{16}{\pi D^3} \sqrt{(2.2073)^2 + (6.8209)^2}$$

$$D = 0.09327 \text{ m}$$

$$D = 93.27 \text{ mm}$$

Solid shaft

$$S_s = \frac{16T}{\pi D_s^3}$$

Hollow shaft

$$S_s = \frac{16TD}{\pi(D^4 - d^4)}$$

torqueing;

$$\frac{16T}{\pi D_s^3} = \frac{16TD}{\pi(D^4 - d^4)}$$

$$\frac{1}{(2.5 \text{ in})^3} = \frac{3.5 \text{ in}}{[(3.5 \text{ in})^4 - d^4]}$$

$$d = 3.125 \text{ in}$$

Short Start ;

$$P = \frac{D^3 N}{30}$$

$$45 \text{ hp} = \frac{(1.8 \text{ in})^3 N}{30}$$

$$N = (293.2099 \text{ rpm}) \left(\frac{1 \text{ min}}{60 \text{ s}} \right)$$

$$N = 4.8868 \text{ rps}$$

Main Shaft

$$P = \frac{D^3 N}{80} = \frac{\left[(60 \text{ mm}) \left(\frac{1 \text{ in}}{25.4 \text{ mm}} \right) \right]^3 (280 \text{ rpm})}{80}$$

$$P = 46.1339 \text{ hp}$$

Line Starts

$$P = \frac{D^3 N}{53.5}$$

$$(180 \text{ kW}) \left(\frac{1 \text{ hp}}{0.746 \text{ kW}} \right) = \frac{D^3 \left(\frac{20 \text{ rev}}{\text{sec}} \right) \left(\frac{60 \text{ sec}}{1 \text{ min}} \right)}{53.5}$$

$$D = 2.2075 \text{ in}$$

$$S_s = \frac{16T}{\pi D^3} = \frac{16 (25000 \text{ in-lb})}{\pi (3 \text{ in})^3}$$

$$S_s = 4715.7020 \text{ psi}$$

$$\theta = \frac{TL}{JG} ; \quad \delta_s = \frac{16T}{\pi D^3} ; \quad J = \frac{\pi D^4}{32}$$

equating $T = T$

$$\frac{G\theta J}{L} = \frac{\pi \delta_s D^3}{16}$$

$$\frac{G\pi\theta D^4}{32L} = \frac{\pi \delta_s D^3}{16}$$

$$D = \frac{2\delta_s L}{\theta G} = \frac{2(80 \text{ MPa})(5 \text{ m})}{(5^\circ)\left(\frac{\pi}{180^\circ}\right)(90 \times 10^3 \text{ MPa})}$$

$$D = 0.1019 \text{ m}$$

$$D = 101.9 \text{ mm}$$

$$\theta = \frac{TL}{JG}$$

$$(4 \text{ deg}) \left(\frac{\pi}{180^\circ} \right) = \frac{(15 \text{ kN-m})(8 \text{ m})}{J(85 \times 10^6 \text{ kPa})}$$

$$J = 2.0222 \times 10^{-5} \text{ m}^4$$

for solid shaft:

$$J = \frac{\pi D^4}{32}$$

$$2.0222 \times 10^{-5} \text{ m}^4 = \frac{\pi D^4}{32}$$

$$D = 0.1198 \text{ m}$$

$$D = 119.8 \text{ mm}$$

$$\frac{\theta}{L} = \frac{T}{JG} = \frac{F(\ell/2)}{JG}$$

$$J = \frac{\pi \phi^4}{32} = \frac{\pi \left[2.5 \text{ in} \left(\frac{1 \text{ ft}}{12 \text{ in}} \right) \left(\frac{1 \text{ m}}{3.28 \text{ ft}} \right) \right]^4}{32}$$

$$J = 1.5979 \times 10^{-6} \text{ m}^4$$

then;

$$\frac{\theta}{L} = \frac{(6 \text{ kN}) \left(\frac{1}{2} \text{ ft} \right) \left(\frac{1 \text{ m}}{3.28 \text{ ft}} \right)}{(1.5979 \times 10^{-6} \text{ m}^4) (83 \times 10^6 \text{ kPa})}$$

$$\frac{\theta}{L} = \left(6.8964 \times 10^{-3} \frac{\text{rad}}{\text{m}} \right) \left(\frac{180^\circ}{\pi} \right)$$

$$\frac{\theta}{L} = 0.3951 \text{ deg/m}$$

$$S_s = \frac{16T}{\pi D^3}$$

$$28000 \text{ KPa} = \frac{16(2 \text{ kN-m})}{\pi D^3}$$

$$D = 0.07139 \text{ m}$$

then;

$$S_s = \frac{T_c}{J} = \frac{T D}{2J}$$

$$28000 \text{ KPa} = \frac{(2 \text{ kN-m})(0.07139 \text{ m})}{2J}$$

$$J = 2.5495 \times 10^{-6} \text{ m}^4$$

$$S_s = \frac{16TD}{\pi(D^4 - d^4)} ; P = \frac{2\pi TN}{60}$$

$$120 \text{ kW} = \frac{2\pi T (700 \text{ rpm})}{60}$$

$$T = 1.6370 \text{ kN-m}$$

subst;

$$S_s = \frac{16(1.6370 \text{ kN-m})\left(\frac{1000 \text{ N}}{1 \text{ kN}}\right)\left(\frac{1000 \text{ mm}}{1 \text{ m}}\right)(110 \text{ mm})}{\pi(110^4 - 90^4) \text{ mm}^4}$$

$$S_s = 11.3503 \text{ MPa}$$

$$S_s = \frac{16 T D}{\pi (D^4 - d^4)}$$

$$50\,000 \text{ kPa} = \frac{16 (6 \text{ kN-m}) (0.090 \text{ m})}{\pi (0.090^4 - d^4) \text{ m}^4}$$

$$d = 0.0571 \text{ m}$$

$$d = 57.10 \text{ mm}$$

$$\theta = \frac{584TL}{D^4G} ; L = 20D ; T = \text{in-lb}$$

$$G = \text{psi}$$

$$\theta = \text{deg}$$

$$\theta = \frac{584T(20D)}{D^4G} = \frac{11680T}{D^3G}$$

$$1 \text{ deg} = \frac{11680 (2.5 \text{ kW-m}) \left(\frac{778 \text{ ft-lb}}{1.055 \text{ kW-m}} \right) \left(\frac{12 \text{ in}}{1 \text{ ft}} \right)}{D^3 \left(80 \times 10^6 \text{ kPa} \right) \left(\frac{14.6 \text{ psi}}{101.325 \text{ kPa}} \right)}$$

$$D = (2.8196 \text{ in}) \left(\frac{25.4 \text{ mm}}{1 \text{ in}} \right)$$

$$D = 71.61791 \text{ mm}$$

alternate solution:

Shear deflection of 1 deg for a length 20 times its diameter (for steel)

$$D = 0.10 \sqrt[3]{T}$$

$$D = 0.10 \sqrt[3]{(2.5 \text{ kW-m}) \left(\frac{778 \text{ ft-lb}}{1.055 \text{ kW-m}} \right) \left(\frac{12 \text{ in}}{1 \text{ ft}} \right)}$$

$$D = (2.8073 \text{ in}) \left(\frac{25.4 \text{ mm}}{1 \text{ in}} \right)$$

$$D = 71.3054 \text{ mm}$$

$$S_s = \frac{16T}{\pi D^3} ; \quad hp = \frac{2\pi TN}{33000}$$

$$60hp = \frac{2\pi T(1500 \text{ rpm})}{33000}$$

$$T = 210.0845 \text{ lb-ft}$$

Subst;

$$550 \frac{\text{lb}}{\text{in}^2} = \frac{16 \left(210.0845 \text{ lb-ft} \right) \left(\frac{12 \text{ in}}{1 \text{ ft}} \right)}{\pi D^3}$$

$$D = 2.8579 \text{ in}$$

$$S_s = \frac{16T}{\pi D^3}$$

$$30 \times 10^3 \frac{\text{N}}{\text{m}^2} = \frac{16T}{\pi (0.064\text{m})^3}$$

$$T = 1.5442 \text{ kN-m}$$

Then;

$$P = \frac{2\pi TN}{60}$$

$$80 \text{ kW} = \frac{2\pi (1.5442 \text{ kN-m}) N}{60}$$

$$N = 494.7323 \text{ rpm}$$

$$S_d = \frac{S_y}{F.S.} ; S_s = \frac{16TD}{\pi(D^4 - d^4)}$$

$$\frac{16TD}{\pi(D^4 - d^4)} = \frac{S_y}{F.S.}$$

$$\frac{16(4000 \text{ N-m})\left(\frac{1000 \text{ mm}}{1 \text{ m}}\right)(90 \text{ mm})}{\pi(90^4 - d^4) \text{ mm}^4} = \frac{380 \text{ MPa}}{4}$$

$$d = 82.4935 \text{ mm}$$

$$S_s = \frac{16T}{\pi D^3} ; P = 2\pi TN$$

$$120 \text{ kW} = 2\pi T \left(6 \frac{\text{rev}}{\text{sec}} \right)$$

$$T = 3.1831 \text{ kN-m}$$

Subst;

$$100 \times 10^3 \frac{\text{kN}}{\text{m}^2} = \frac{16 (3.1831 \text{ kN-m})}{\pi D^3}$$

$$D = 0.05453 \text{ m}$$

$$D = 54.53 \text{ mm}$$

$$P = \frac{2\pi T N}{60}$$

$$230 \text{ kW} = \frac{2\pi T (380 \text{ rpm})}{60}$$

$$T = 5.7798 \text{ kN-m}$$

then;

$$T = Fr$$

$$5.7798 \text{ kN-m} = F \left(\frac{0.210 \text{ m}}{2} \right)$$

$$F = 55.0461 \text{ kN}$$