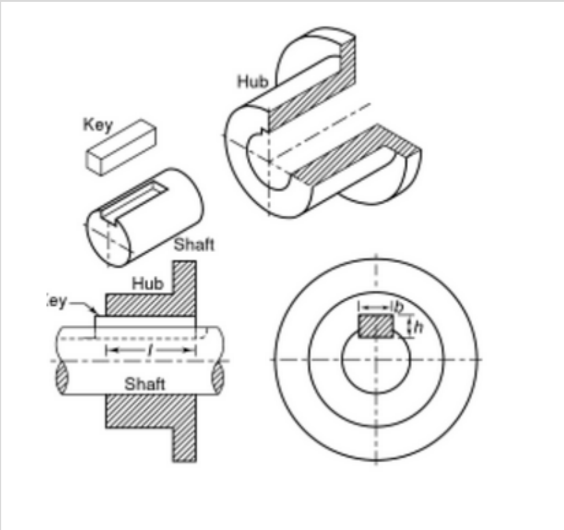
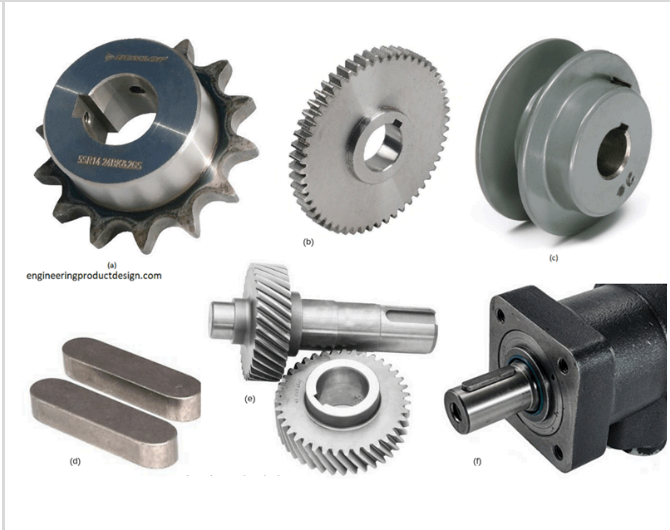
In mechanical systems, the connection between a rotating shaft and a hub is crucial for efficient power transmission. This is commonly achieved using a **shaft, hub, and key assembly**, which ensures that torque is transmitted without slippage between components. This method is widely used in motors, pumps, gear drives, and pulley systems.





The **shaft** must be strong, tough, and wear-resistant because it transmits torque and experiences rotational forces. It is typically made from mild steel, alloy steel, or stainless steel, depending on the application.

The **hub** must be strong enough to support the rotating component, such as a gear, pulley, or coupling, while being slightly softer than the shaft to minimize wear. Cast iron is widely used for its excellent damping properties and ease of manufacturing. In high-torque applications, steel, whether mild or alloy, is preferred for its superior strength. Alternatively, aluminum or brass is used in lightweight applications where corrosion resistance is necessary.

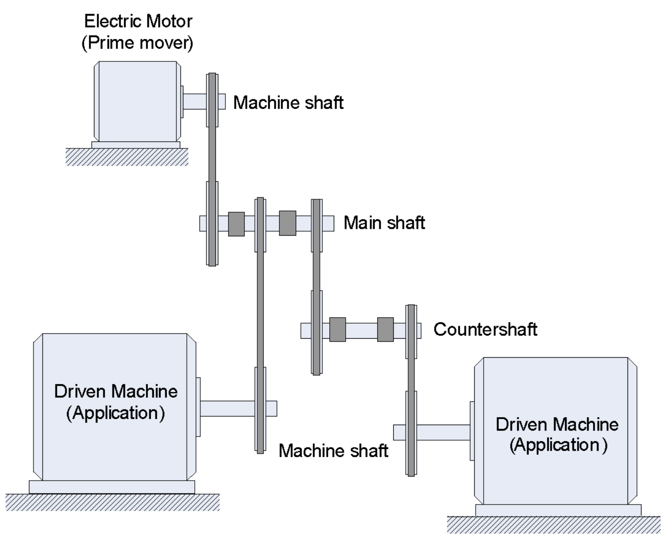
The **key** plays a crucial role in preventing relative motion between the shaft and the hub. It must be softer than the shaft but harder than the hub, allowing it to act as a sacrificial component. In case of an overload, the key deforms or shears first, protecting the more expensive shaft and hub from damage.

SHAFT DESIGN

Shafts are components of a mechanical device supported by bearings that transmits rotational motion (mechanical power) from a motor or an engine to a point of application.

Types:

1. **Transmission (Line) Shaft:** Long shafts connecting prime movers to gears or pulleys.
2. **Countershaft:** Connects the prime mover to the driven machine.
3. **Flexible Shaft:** Used for motion or power transmission with angular flexibility.
4. **Spindle:** Short rotating shafts specific to certain machine tools.



Design Considerations

1. Strength and Rigidity: Shaft design ensures sufficient strength to resist fracture under torsion, bending, and axial loads.
2. Diameter Selection: Larger diameters handle higher loads but must balance cost and material use.
3. Stress Concentration: Avoid abrupt changes in cross-section to minimize stress concentration (e.g., use grooves and fillets).
4. Critical Speed: The natural frequency of the shaft must exceed operating speeds to avoid resonance and vibration issues.

Power Transmitted by Shafts: $P = \frac{2\pi TN}{60}$

Where:
P = power transmitted, kW
T = torque or torsional moment
N = speed (rpm)

Shaft Design:

Shafts can be subjected to (1) torsional, (2) bending, (3) axial loads, or (4) combination of these loads.
If the load is torsional, the stress induced is shear; if bending, the stresses are tension and compression.

Based on Strength

Designing a shaft based on strength ensures that it can withstand the various forces and stresses acting on it without failure or fracture.

1. Pure Torsion

$$S_s = \frac{16\,T}{\pi\,D_o^3} \quad ; \quad \text{Solid shaft}$$

$$S_s = \frac{16\,T\,D_o}{\pi\left(D_o^4 - D_i^4\right)} \quad ; \quad \text{Hollow shaft}$$

2. Pure Bending

$$S_f = \frac{32\,M}{\pi\,D_o^3} \quad ; \quad \text{Solid shaft}$$

$$S_f = \frac{32\,M\,D_o}{\pi\left(D_o^4 - D_i^4\right)} \quad ; \quad \text{Hollow shaft}$$

3. Combination of torsion and bending

Most shafts are installed with pulleys, sprockets, or sheaves that may cause bending in addition to torsion.

For ductile material: (maximum shear stress theory or Tresca's Criterion)

$$S_{s\,max} = \frac{16}{\pi\,D_o^3}\sqrt{M^2 + T^2} \quad ; \quad \text{Solid shaft}$$

$$S_{s\,max} = \frac{16\,D_o}{\pi\left(D_o^4 - D_i^4\right)}\sqrt{M^2 + T^2} \quad ; \quad \text{Hollow shaft}$$

For brittle material: (maximum normal stress theory)

$$S_{t\,max} = \frac{16}{\pi\,D_o^3}\left(M + \sqrt{M^2 + T^2}\right) \quad ; \quad \text{Solid shaft}$$

$$S_{t\,max} = \frac{16\,D_o}{\pi\left(D_o^4 - D_i^4\right)}\left(M + \sqrt{M^2 + T^2}\right) \quad ; \quad \text{Hollow shaft}$$

Note: if the type of material is not given, assume it to be ductile material.

Based on Stiffness

Designing a shaft based on stiffness ensures that it maintains its shape and alignment under operational loads, preventing excessive deformation.

$$S_s = K_t \cdot \frac{16\,T}{\pi\,D_o^{\,3}}$$

$$S_f = K_f \cdot \frac{32\,M}{\pi\,D_o^{\,3}}$$

$$S_{s\,max} = \frac{16}{\pi\,D_o^{\,3}}\sqrt{\left(K_f M\right)^2 + \left(K_t T\right)^2}$$

$$S_{t\,max} = \frac{16}{\pi\,D_o^{\,3}}\left[K_f M + \sqrt{\left(K_f M\right)^2 + \left(K_t T\right)^2}\right]$$

Based on Torsional Rigidity:

$\theta = \frac{TL}{JG}$, radians

Where: T = maximum torque
- M = maximum bending moment
- D_o = outside diameter
- D_i = inside diameter
- S_s = shearing stress
- S_f = flexural stress
- S_{smax} = maximum shearing stress
- S_{tmax} = maximum tensile or compressive stress angle of twist, radians
- K_f = stress concentration factor for maximum bending
- K_t = stress concentration factor for maximum torsion
  - K_f and K_t can be found in Shigley's Mechanical Engineering Design by Richard Budynas, Keith Nisbett
- L = length of shaft
- J = polar moment of inertia
- G = torsional modulus of elasticity
  - For steel: G = 12 x 10^6 psi = 80 GPa

Theoretical relation between shear modulus, G and tensile modulus, E

$$G = \frac{E}{2\,(1-\mu)}$$

Where: μ =Poisson's Ratio, 0.3 is often used for steel

Materials for Shafts:

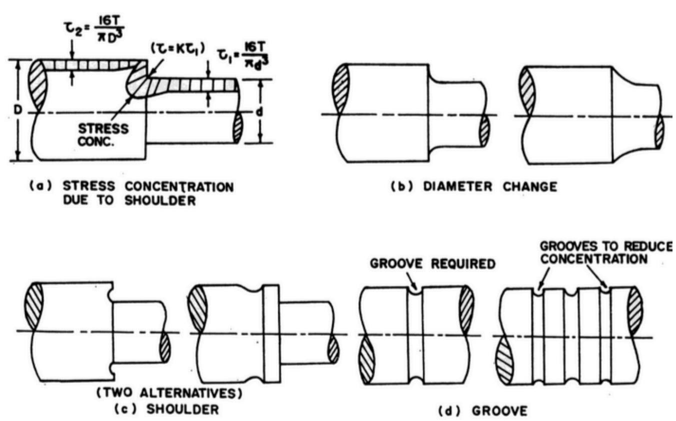
- Common Materials: Plain carbon steel (0.2%-0.5% carbon content).
- Types:
  - Hot-Rolled Steel: Least expensive; requires machining for smooth surfaces.
  - Cold-Rolled Steel: Greater accuracy; available in alloy forms.
  - Alloy Steels: Offer high resistance to shock and wear for severe conditions.
- Heat Treatment: Enhances strength and wear resistance.

Methods to Reduce Stress Concentration:

Smooth or filleted designs eliminate sudden changes in cross-sectional areas, which are prime causes of high stress intensities. Grooves create an even stress flow, ensuring that no specific point bears excessive stress. The reduced concentration lowers the likelihood of crack initiation and propagation, enhancing the shaft's fatigue life and durability.

K_t = stress concentration factor for shafts

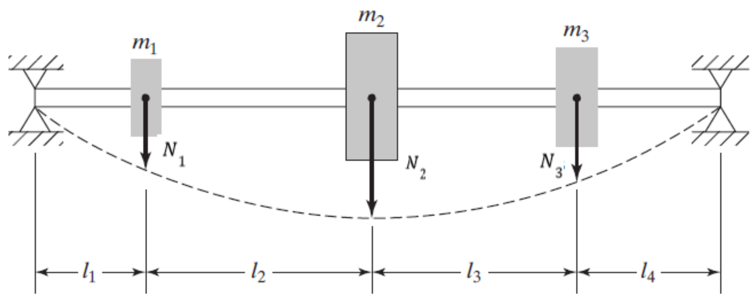
Shigley's Mechanical Engineering Design by Richard Budynas, Keith Nisbett



1. Smooth Transition Designs:
   - Using fillets or curved transitions between different diameters reduces the sharpness of the stress concentration area (as shown in figure (b)).
   - Gradual changes in cross-sectional dimensions prevent abrupt stress peak.
2. Grooves or Relief Features:
   - Adding grooves (figure (d)) spreads the stress over a larger area, minimizing localized stress intensities.
   - Grooves also prevent sharp edges that can act as stress risers.
3. Alternative Shoulders:
   - Replacing sharp shoulders with filleted shoulders (figure (c)) avoids sharp edges, thereby reducing stress buildup.

4. Material Choice and Surface Treatment:
    - Proper material selection with higher ductility or surface hardening through techniques like nitriding helps distribute stress more evenly.
    - Polished surfaces reduce micro-cracks, further decreasing stress concentration.

Dunkerley's Equation is a formula used to estimate the critical speed of a shaft-rotor system, which is the speed at which the system experiences resonance. Resonance can lead to excessive vibrations and potential failure, so it's important to calculate the critical speed to avoid operating at or near this speed.



$$\frac{1}{N_S^{\,2}} = \frac{1}{N_1^{\,2}} + \frac{1}{N_2^{\,2}} + \frac{1}{N_3^{\,2}} + ... + \frac{1}{N_n^{\,2}}$$

Where: N_S = critical Speed of the system, rpm

Charles Dunkerley was a British mechanical engineer (late 19th–early 20th century) who specialized in vibration analysis. He developed a formula that simplifies the calculation of the critical speed of a shaft carrying multiple loads. His equation is a widely used approximation method in rotor dynamics and structural vibration.
Dunkerley's equation is a fast and practical way to calculate shaft critical speeds without solving complex differential equations.

Factors Affecting Critical Speed

- Shaft Length – Longer shafts have lower critical speeds.
- Diameter – Thicker shafts have higher critical speeds.
- Bearing Support – More rigid supports increase the critical speed.
- Mass Distribution – Heavy loads lower the critical speed.

Empirical Formula from Machinery's Handbook 29th Edition:
By Erik Oberg, Franklin D. Jones, Horlbrook L. Horton, and Henry H. Ryffel, Page 295

Shaft diameter:

1. For allowable twist not exceeding 0.08 deg per ft length

$D = 0.29\sqrt[4]{T}$ or $D = 4.6\sqrt[4]{\frac{HP}{N}}$

2. For allowable twist not exceeding 1 deg per 20D length

$D = 0.1\sqrt[3]{T}$ or $D = 4.0\sqrt[3]{\frac{HP}{N}}$

Where: D = diameter of the shaft, in
T = torque, in-lb
HP = horsepower transmitted, hp
N = shaft speed, rpm

Distance between bearings:

1. For shafts subjected to no bending action except its own weight

$$L = 8.95\sqrt[3]{D_o^{\ 2}}$$

2. For shafts subjected to bending action of pulleys, sprockets, sheaves, etc.

$$L = 5.20\sqrt[3]{D_o^{\ 2}}$$

Where: L = maximum distance between bearings, ft
D_o = diameter of the shaft, in

Note: Pulleys should be placed near the bearings as possible and if the shaft is up to three inches in diameter, it is suggested to use cold-rolled steel.

Shaft Power Transmission Systems: $P = \frac{D^3N}{C}$

For main power transmitting shafts (assumed allowable stress = 4000 psi): $P = \frac{D^3N}{80}$

For line shafts carrying pulleys (assumed allowable stress = 6000 psi): $P = \frac{D^3N}{53.5}$

For small, short shafts (assumed allowable stress = 8500 psi): $P = \frac{D^3N}{38}$

Where: P = power transmitted, HP
D = outside diameter of shaft, inches
N = speed, rpm