Step 1: Conversion of Shaft Parameters to Consistent English Units

To prevent the unit conversion errors present in the handwritten sheet, we will convert the shaft diameter (d) from millimeters to inches. Using the exact conversion factor (1 in = 25.4 mm):

$$d = (80\ mm)\left(\frac{1\ in}{25.4\ mm}\right) = 3.1496\ in$$

The yield point of the SAE 1040 cold-rolled shaft (S_y) is given as 55 ksi = 55,000 psi.

Step 2: Determine the Maximum Torque Capacity (T) of the Shaft

According to the maximum shear stress theory (often applied in mechanical board exams when designing shafts under pure torsion), the allowable shear stress (S_{all}) is related to the tensile yield strength (S_y) by a factor of 0.6 divided by the factor of safety (FS):

$$S_{all} = \frac{0.60\ S_y}{FS}$$

Substituting the shaft properties (S_y = 55,000 psi and FS = 5):

$$S_{all} = \frac{0.60\ (55{,}000\ psi)}{5.0} = 6,600\ psi$$

Using the solid shaft torsion equation, we determine the maximum design torque (T) the shaft can safely transmit:

$$S_{all} = \frac{16T}{\pi d^3}$$

$$T = \frac{\pi S_{all} d^3}{16} = \frac{\pi(6{,}600\ psi)(3.1496\ in)^3}{16} = 40,404.14\ lb - in$$

Step 3: Compute the Key's Required Yield Point ($S_{y,\ key}$) Based on Crushing

The allowable compressive bearing stress ($S_{c,\ all}$) for the key interface ensures that the projecting side face does not crush under load. The formula for compressive failure relates torque, diameter, thickness, and length as follows:

$$S_{C,\,all} = \frac{4T}{dtL}$$

Because the problem asks for the minimum yield point of the key material using a factor of safety of 5, we set $S_{C,\,all} = \frac{S_{y,\,key}}{FS}$:

$$\frac{S_{y,\,key}}{FS} = \frac{4T}{dtL}$$

$$\frac{S_{y,key}}{5} = \frac{4(40{,}404.14\ lb-in)}{(3.1496\ in)(0.75\ in)(4\ in)}$$

$$S_{y,\,key} = 85{,}522.25\ psi$$

The shaft material yields at 55ksi, but the key requires a much tougher material yielding at roughly 85.6 ksi. This happens because the key has a relatively small profile (t = 0.75 in) relative to the overall diameter of the shaft. Since the forces are concentrated over a narrow bearing area, the key must be made of a stronger material to prevent it from crushing.

Real-world components are typically optimized so that the key is equally strong in both shear and compression. For a square key where b = t, this balance occurs naturally when the allowable shear limit is half the compressive limit. For this rectangular key, the mismatched proportions (b=0.5in vs. t=0.75in) mean an engineer must check both failure modes independently to ensure the key shears cleanly before it deforms or ruins the shaft's internal keyway.