

Step 1: Calculate the Transmitted Torque (T)

The first objective is to compute the mechanical rotational torque generated by transmitting a given power at a specific angular velocity. The problem provides the system power in kilowatts ($P = 120 \text{ kW}$) and the operational speed in revolutions per minute ($N = 1800 \text{ rpm}$). To use standard imperial formulas, we first convert the power from kilowatts to horsepower (HP):

$$HP = (120 \text{ kW}) \left(\frac{1 \text{ hp}}{0.746 \text{ kW}} \right) = 160.8579 \text{ hp}$$

Using the fundamental relationship for imperial mechanical power, we isolate the torque (T) in units of pound-in (lb-in):

$$HP = \frac{2\pi TN}{33,000}$$

$$160.8579 \text{ hp} = \frac{2\pi T(1800 \text{ rpm})}{33,000}$$

$$T = (469.3577 \text{ lb} - \text{ft}) \left(\frac{12 \text{ in}}{1 \text{ ft}} \right) = 5,632.2924 \text{ lb} - \text{in}$$

Step 2: Determine Spline Geometric Characteristics

The shaft has an outer nominal major diameter of $D = 2.0 \text{ in}$. Based on the proportional relationships defined for a 10-spline permanent fit:

- **Minor (Root) Diameter (d):**

$$d = 0.91D = 0.91(2.0 \text{ in}) = 1.82 \text{ in}$$

- **Spline Width (b):**

$$b = 0.156D = 0.156(2.0 \text{ in}) = 0.312 \text{ in}$$

- **Hub Engagement Length (L):** Given as 1.75 in ($1\text{-}\frac{3}{4}$ in)
- **Number of Splines (N_s):** Given as 10

Step 3: Compute the Spline Shearing Stress (S_s)

The shearing stress in a spline profile is experienced along the base of each spline tooth where it intersects the main core of the shaft. The total resisting shear area (A_s) provided by all splines working in parallel is equal to the width of a single spline tooth multiplied by the hub engagement length, scaled up by the total number of splines:

$$A_s = bLN_s$$

The nominal shear force (F_s) acting at the minor radius $\left(\frac{d}{2}\right)$ is derived from the torque $\left(T = F_s \frac{d}{2}\right)$. Incorporating the 1.1 load-distribution factor required by the reference material, the standard formula becomes:

$$S_s = \frac{2T (1.1)}{dbLN_s}$$

Substituting the converted imperial parameters into the equation:

$$S_s = \frac{2(5,632.2924 \text{ lb-in}) (1.1)}{(1.82 \text{ in})(0.312 \text{ in})(1.75 \text{ in})(10)} = 1,246.9350 \text{ psi}$$

While a standard key concentrates all of its driving force onto one or two points on a shaft's circumference (creating high localized stress), a multi-spline shaft distributes the torque evenly across its entire perimeter. This arrangement results in a very low shear stress (**1,246.9 psi**), making splines ideal for heavy-duty industrial designs like automotive drivetrains, aerospace gearboxes, and power take-off (PTO) systems.

In a perfect theoretical model, every single spline carries an identical portion of the mechanical load. However, due to minor manufacturing tolerances, slight tool wear, or indexing misalignments during machining, some teeth will contact the hub slightly before others. Mechanical designers use the **1.1 factor** to add a 10% safety margin, ensuring the assembly remains safe even if the load is not perfectly distributed among all 10 splines.