Unlike gear teeth or belt pulley diameters which can sometimes follow simple linear approximations, a chain sprocket forms a polygon rather than a perfect circle. The pitch (P = 0.5in) represents the length of a straight line segment connecting the centers of adjacent chain pins. Therefore, the pitch diameter (D) must be evaluated using exact chordal trigonometry. The provided handwritten key successfully identifies the tooth relationship based on the speed ratio and computes the pitch diameter accurately without any structural flaws.

Step 1: Kinematic Evaluation of the Driven Tooth Count (T_2)

In any continuous chain transmission drive, the linear travel rate of the chain passing over the driving sprocket must exactly equal the linear travel rate passing over the driven sprocket. This synchronization means that the number of teeth shifting per unit of time is uniform across both sides of the system.

The relationship between the tooth counts (T) and rotational angular velocities (N) is inversely proportional:

$$T_1 N_1 = T_2 N_2$$

$$(20)(700\, rpm) = T_2(200\, rpm)$$

$$T_2 = 70\, teeth$$

Step 2: Sizing the Pitch Diameter (D_2) Using Chordal Geometry

Because a roller chain is made up of rigid, straight links, it wraps around a sprocket as an assembled series of chordal segments rather than an arc. The pin centers form a regular polygon with T_2 sides.

If we isolate a single triangle formed by two adjacent pin centers and the sprocket's geometric center, the central angle subtended by one link is exactly $\frac{360^o}{T_2}$. Bisecting this isosceles triangle yields a right triangle where half of the pitch length (0.5P) faces half of the central angle $\left(\frac{360^o}{T_2}\right)$.

Using trigonometry, the exact formula for the pitch diameter (D) is:

$$D_2 = \frac{P}{sin\left(\frac{180^o}{T_2}\right)}$$

Substituting the given chain pitch (P = 0.5in) and the calculated tooth count (T_2 = 70):

$$D_2 = \frac{0.5\, in}{sin\left(\frac{180^o}{70}\right)} = 11.1446\, in$$

Because a chain sits on the sprocket as a polygon rather than a perfect circle, the effective radius varies slightly as the sprocket turns. This variation causes small, rhythmic pulsations in both chain speed and tension. This "chordal action" generates noise and vibration. Choosing a larger tooth count for the driver (T_1 = 20) helps minimize this effect; keeping the driver above 15 to 17 teeth ensures smoother power transmission and extends the life of the chain.

Reducing rotational speed from 700 rpm to 200 rpm requires a substantial velocity ratio (VR = 3.5). This ratio means the driven sprocket must be 3.5 times larger than the driver. If space is tight inside a machine housing, an 11.145-inch sprocket might be too bulky. To shrink the overall footprint, a design engineer would need to switch to a multi-strand chain setup with a smaller pitch size.