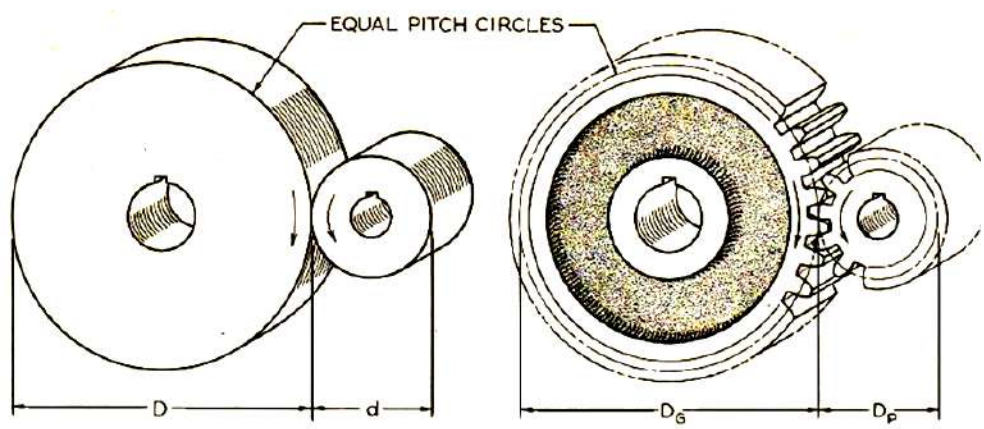
Power transmission systems often rely on various mechanical components to transfer motion and torque between shafts. Among these, belts and friction pulleys are common but are prone to slippage due to their reliance on friction. This limitation makes them unsuitable for applications requiring precise and invariable speed ratios. In contrast, chains and gears provide positive drives, meaning they can transmit motion without slippage. For applications where the center distance between shafts is relatively short and a reliable transfer of power is essential, toothed gears, especially spur gears, are the preferred choice.

Spur gears are toothed wheels with radially arranged teeth parallel to the shaft axis. They are used to transmit motion and power from one shaft to another parallel shaft.

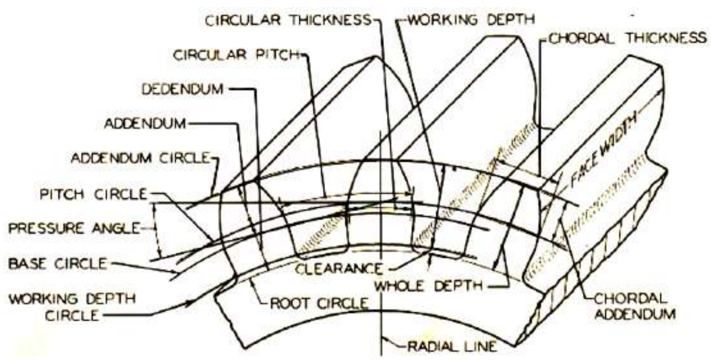


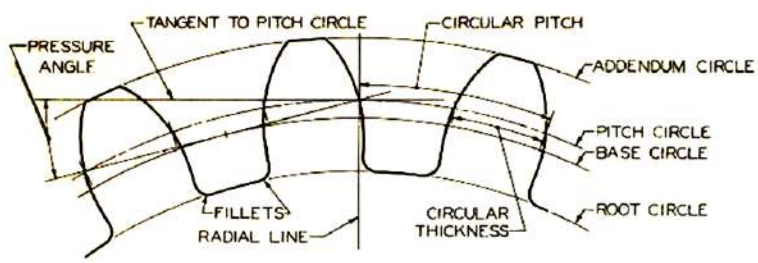
A variation of the spur gear is the **internal gear**. In this configuration, the teeth are cut on the inner surface of a cylindrical ring and mesh with a smaller external gear or pinion. This arrangement is compact and often used in planetary gear systems.

Another example is the **stepped gear**, which combines multiple gears of different sizes into one component, allowing for different speed ratios within a single assembly.

Definitions:

Understanding the nomenclature of spur gears is crucial for their design and application.





Pitch circle

An imaginary circle that corresponds to the circumference of the friction gear from which the spur gear is derived.

Pitch diameter, D

The diameter of the pitch circle of gear or pinion

Circular Pitch, P_C

The distance measured along the pitch circle from a point on one tooth to the corresponding point on the adjacent tooth

$$P_C = \pi \frac{D}{T}$$

Diametral Pitch, P_d

A ratio equal to the number of teeth on the gear per inch of pitch diameter

$$P_d = \frac{T}{D}$$

Note: $P_C P_d = \pi$

Module, m

Module is a metric measurement that defines the ratio of pitch diameter, in millimeters to the number of teeth.

$$m = \frac{D\ (in\ mm)}{T}$$

Module and diametral pitch relationship:

$$m = \frac{25.4}{P_d}$$

P_d = in inches

m = in millimeters

Circular Pitch, P_C

P_C = space width + tooth thickness

P_C = S + T

Backlash = space width – tooth thickness

B = S – T

Substituting;

P_C = B + 2T

If backlash, B is not given, use: $B = \frac{0.035}{P_d}$

Face width, f

If face width is not given, use: $f = 3P_C$ or $f = \frac{10}{P_d}$

Conditions for interchangeable gear tooth:

a. All gears have the same pitch
b. All gears have the same addendum
c. All gears are cut with the same angle of obliquity

Proportions for Interchangeable Gear Tooth System:

	Full-depth Involute system	**Stub tooth system (ϴ=20º)**
Addendum, a	$a = \frac{1}{P_d} = \frac{P_c}{\pi}$	$a = \frac{0.80}{P_d} = \frac{0.80P_c}{\pi}$
Clearance, c	$c = \frac{0.25}{P_d}$	$c = \frac{0.20}{P_d}$
Dedendum, b	$b = \frac{1.25}{P_d}$	$b = \frac{1}{P_d} = \frac{P_c}{\pi}$
Working depth, WD	$WD = \frac{2}{P_d}$	$WD = \frac{1.6}{P_d}$
Total depth or whole depth, TD	$TD = \frac{2.25}{P_d}$	$TD = \frac{1.80}{P_d}$

Outside diameter, D_o

$$D_o = D + 2a$$

Root diameter, D_R

$$D_R = D - 2b$$

Pressure angle, α

The angle that determines the direction of pressure between contacting teeth, and designates the shape of the involute teeth. (14.5º or 20º involute teeth)

Base circle

The circle from which the involute profile is generated.

Center-to-center distance between shafts, C:

$$C = \frac{D_2 + D_1}{2}$$

Velocity Ratio,

$$VR = \frac{N_1}{N_2} = \frac{D_2}{D_1} = \frac{T_2}{T_1}$$

Where:

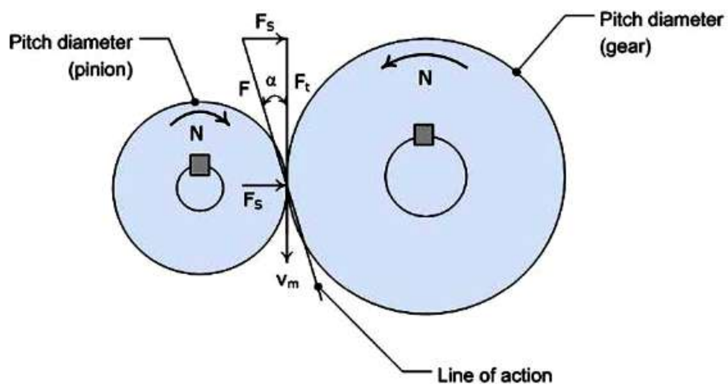
D_1 and D_2 = pitch diameters of smaller gear (pinion) and larger gear (gear), respectively.

T_1 and T_2 = number of teeth of smaller gear (pinion) and larger gear (gear), respectively.

N_1 and N_2 = the revolution per minute of smaller gear (pinion) and larger gear (gear), respectively.

Pressure angle, α

In the geometry of spur gears, two critical elements are the base circle and the pressure angle. The base circle is the fundamental circle from which the involute profile of a gear tooth is generated. An **involute** is the curve traced by a point on a taut string as it unwinds from the circumference of a base circle. This involute curve defines the shape of the gear tooth profile used in most modern gears due to its unique ability to maintain a constant velocity ratio during meshing.



Where:

F = load on the line action or tooth load or total normal load, This is the actual, total force exerted by one tooth onto the mating tooth along the line of action. In a right triangle

$$F = \frac{F_t}{\cos \alpha} = \frac{F_S}{\sin \alpha}$$

F_S = separating load or radial load. This load points directly toward the center (axis) of the gear to separate them, this is the force that causes the shaft to "bend."

F_t = tangential or transmitted load. This the force component that directly turns the gear and generates Torque

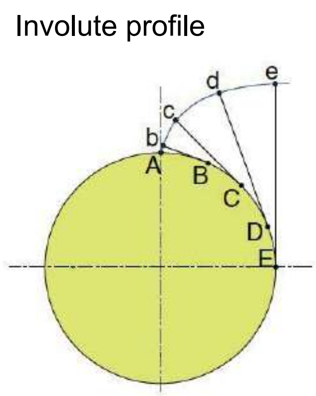
v_m = pitch line velocity

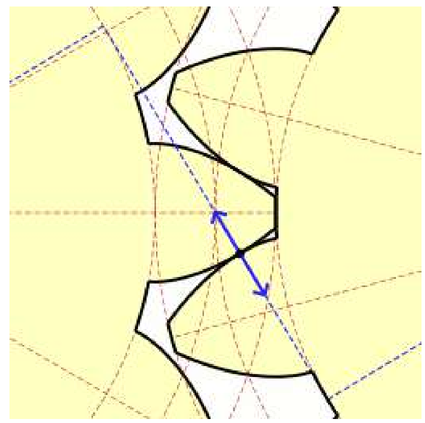
$$v_m = \pi D_1 N_1 = \pi D_2 N_2$$

Note: If pressure angle is not given, let it be 14.5°, for a full depth tooth system.

Law of Gearing and the Action of Gear Teeth

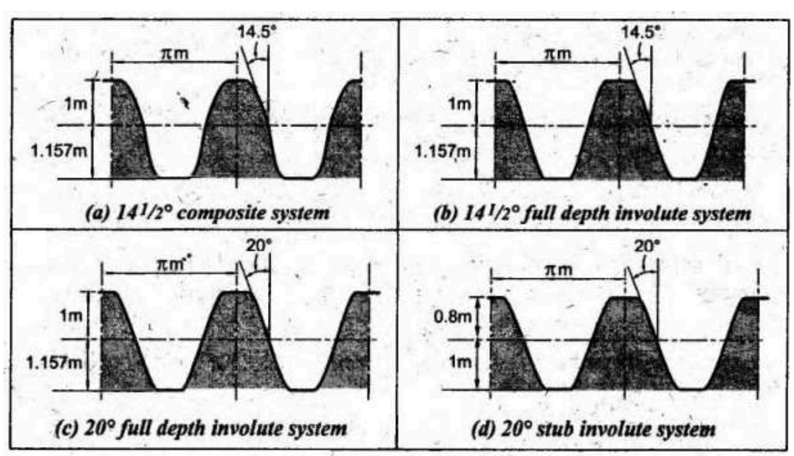
The **law of gearing** is fundamental in ensuring that gears transmit motion with a constant velocity ratio. This law states that *the common normal at the point of contact between two meshing teeth must always pass through the pitch point*. The pitch point is the point of tangency between the pitch circles of the mating gears. When this condition is satisfied, the gears maintain a uniform speed ratio throughout their engagement.





It's important to note that the law of gearing pertains to the instantaneous velocity ratio during the meshing of teeth, not the overall transmission ratio. If the velocity ratio fluctuates during engagement, it leads to vibration and noise due to sudden accelerations or decelerations. Therefore, to ensure smooth and silent operation, the gear tooth profiles must be conjugate curves, which inherently comply with the law of gearing. **Involute profiles** are the most common example of such conjugate curves.

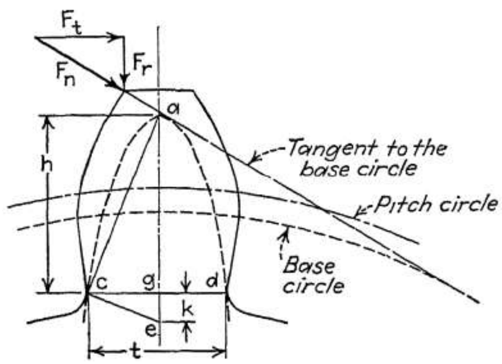
Types of Tooth Profile



The figures show three involute gear systems: 14.5° full-depth, 20° full-depth, and 20° stub. The 14.5° full-depth system has taller, slender teeth that run smoothly but are weaker and prone to undercutting. The 20° full-depth system, now standard, produces stronger teeth with reduced undercut risk and good overall load capacity. The 20° stub system uses shorter, stouter teeth that give maximum strength and durability for heavy-duty applications, though with a lower contact ratio.

Tangential Force Transmitted at the Pitch Line:

Lewis Equation estimates the bending strength of a gear tooth (spur or straight bevel, preliminarily) by modeling a tooth as a cantilever beam loaded at its tip. It tells you the allowable tangential load F_t a tooth can carry—or, conversely, the bending stress produced by a given F_t.



Generally, the Lewis equation is used for finding the endurance load (static capacity) of a gear tooth based on material strength, tooth geometry, and stress concentration.

Lewis Equation: $F_t = \frac{S_W f Y}{K_f P}$

Where:

$S_W = 0.40 S_U$ = safe working stress

The working stress in gear teeth is the allowable safe stress used in the Lewis equation to determine tooth load capacity. Because actual gears are not perfect beams, and because of tooth inaccuracies, elastic deflections, and dynamic effects at high velocities, the theoretical Lewis load is reduced by a velocity factor.

Use Table AT6 of Design of Machine Elements 4th Edition by Virgil Faires page 570 to 571. Or refer to Table 80, Safe beam stress or static stress of materials for gear, page 382 from Design of Machine Members 2nd Edition by Venton Levy Doughtie and Alex Vallance

K_f = fatigue stress concentration factor at the gear root, which reduces allowable tangential load. The value of K_f should fall between 1.2 to 1.7, when the load is applied in the tip, but higher when the load is applied near the middle, say 1.4 to 2.

Y = Lewis' form factor. Use Table AT24, Form factors for the Lewis' equation, page 601 from Design of Machine Elements 4th Edition by Virgil Faires. Typical Lewis factor for the expected tooth count range (≈ 0.3–0.35 for pinions with 12–20 teeth).

Dynamic Loads on Gear Teeth:

Dynamic load in gears refers to the total maximum instantaneous load acting on gear teeth, which is greater than the transmitted tangential load F_t. It arises from tooth inaccuracies, elastic deflections, periods of acceleration, inertia forces, high-speed impacts, and other similar effects that introduce additional stresses during meshing.

$$F_d = F_t + F_i$$

Where: F_i = incremental load (variable load), in lbs
accounts for vibration, pitch error, tooth inaccuracies.

$$F_i = \frac{0.05v\left(K\cdot f + F_t\right)}{0.05v + \sqrt{K\cdot f + F_t}}$$

K = factor depending upon machining errors
Use Table 82, Maximum error in action between gears, page 385 and Table 83, Values of the dynamic load factor, K, page 386, from Design of Machine Members 2nd Edition by Venton Levy Doughtie and Alex Vallance

Buckingham Equation

The Buckingham equation is the fundamental method for calculating dynamic load on gear teeth, accounting for velocity and tooth errors. (Author of the Analytical Mechanics of Gears) The simplified velocity factor equations shown here are practical approximations of Buckingham's work, classified by gear quality: commercial, carefully cut, and precision cut. They are widely used when detailed error constants are not available.

For ordinary industrial gears operating at velocities up to 2,000 fpm

$$F_d = F_t\left(\frac{600 + v}{600}\right)$$

For accurately cut gears operating at velocities up to 4,000 fpm

$$F_d = F_t\left(\frac{1,200 + v}{1,200}\right)$$

For precision gears cut with a high degree of accuracy and operating at velocities up to 4,000 fpm and over.

$$F_d = F_t\left(\frac{78 + \sqrt{v}}{78}\right)$$

The tangential force F_t at the pitch line:

$$F_t = \frac{(33,000)HP}{v}$$

v = pitch line velocity, in fpm
f = face width, in inches
P = Diametral pitch
F_t = tangential force transmitted, lbs

If the gear is to be keyed to a shaft, the maximum permissible bore can be determined using:

$$d = D\left(0.50 + 0.0344\sqrt{T - 12}\right)$$

Where: d = maximum permissible bore
D = pitch diameter
T = number of teeth

GEAR TOOTH WEAR

Gear-tooth wear is a critical concern in the operation and longevity of gear systems. While surface fatigue failure, commonly known as **pitting**, is often highlighted, other forms of wear significantly affect gear performance and reliability. These include **abrasion**, **scoring**, **spalling**, and **corrosion**, each resulting from different operating conditions and lubrication failures.

Abrasion is typically caused by the presence of foreign materials, such as grit or metal particles, which enter the gear system due to insufficient sealing or dirty lubricants. It may also occur from a failure of the oil film at low speeds. To minimize abrasion, protective enclosures and the use of clean, high-quality lubricants are essential.

Scoring, on the other hand, arises when the oil film breaks down under conditions of high speed and load, causing direct metal-to-metal contact. In such cases, the surface metal may partially melt and smear across the gear teeth, damaging the profile. Scoring is often associated with thin oil films and insufficient extreme pressure (EP) additives in lubricants. These additives form a protective layer that prevents welding and smearing under severe conditions. Scoring is most likely to occur where sliding motion is highest, particularly near the tooth tips. It poses significant issues in hardened gears and can lead to rapid tooth failure if not addressed.

Spalling is a more severe surface fatigue phenomenon than pitting, characterized by the removal of larger flakes of material. This type of wear generally affects hardened gears and can be especially damaging due to the sudden and extensive loss of tooth material.

Corrosion weakens gear teeth by deteriorating the material surface due to chemical reactions, usually in the presence of moisture or corrosive substances. Although corrosion is less of a problem in well-lubricated and enclosed systems using clean petroleum oils, it must be addressed in harsh operating environments to prevent long-term damage.

MATERIALS USED FOR GEARS

Gears are manufactured from a wide range of materials depending on their application, cost, strength, and wear resistance requirements. The most basic and economical material is **ordinary gray iron**, such as ASTM 20, which is adequate for light-duty applications. More frequently, **carbon and alloy steels** are used, with common choices including ASTM classes 30, 40, and 60. High-strength steels like class 60 or higher are suited for demanding applications when proper heat treatment is applied.

Wrought-steel gears, while inexpensive and possessing low wear capacity due to their carbon content (typically 0.30 - 0.50%), can be improved significantly through heat treatment. Small gears are often machined from cold-finished stock and require annealing to relieve stresses. **Cast steel**, though more distortion-prone, is also used, especially when improved mechanical properties are needed through **oil quenching** or **normalizing**. The **surface hardness** for gears is usually increased to values like 350–450 BHN through heat treatment to enhance wear resistance and load capacity.

In more demanding situations, **alloy steels** such as 1335, 5140, 4037, and 8620 are utilized. These steels vary in hardenability and are heat treated through methods such as **carburizing**, **flame hardening**, **induction hardening**, and **nitriding**. For instance, **8620 steel** is widely used for carburized gears and can reach high surface hardness levels without excessive brittleness. The **depth of case hardening** is critical to performance, often being around 0.035–0.05 inches for optimal fatigue resistance.

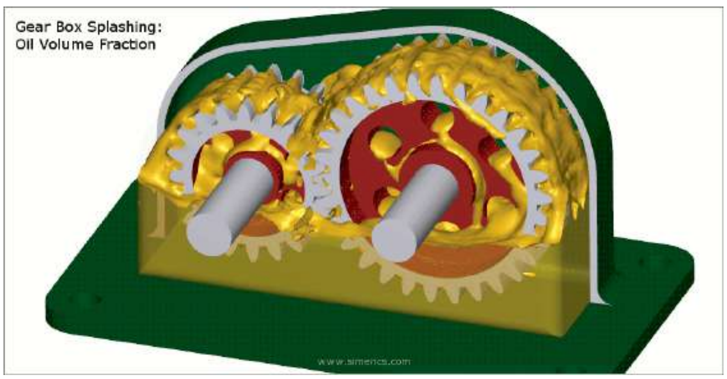
When through-hardening is necessary for high-load applications, steels with a carbon content of at least 0.35% are used. These processes increase hardness throughout the gear body but may compromise machinability. As such, machining is often done before heat treatment, except when high-precision is required after hardening.

For automotive gears and those subjected to continuous operation, **case-hardened** components provide a tough core with a hard, wear-resistant surface. Additional processes like **peening** may be applied after heat treatment to further improve fatigue life and reduce surface stress.

In summary, the choice of gear material and its heat treatment is a balance between cost, performance, machinability, and operating conditions. Proper selection ensures that gears maintain their strength, resist wear, and operate efficiently over long service periods.

LUBRICATION OF GEAR TEETH

Proper lubrication is one of the most essential factors in ensuring the longevity and efficient operation of gear systems. To obtain maximum service life, gears must be supplied with an ample and continuous supply of a suitable **lubricant**. The primary role of the lubricant is to maintain a protective **oil film** between meshing teeth. This film prevents direct metal-to-metal contact, thereby reducing friction, wear, and heat generation—particularly in the **pinion**, which engages more frequently and heats up faster than the larger mating gear.



The lubricant must strike a balance: it should be **thin enough** to flow easily into the narrow gaps between gear teeth, yet **thick and strong enough** to withstand the high pressures at the tooth contacts without breaking down. If the oil film fails, **abrasion** or **scoring** can occur, especially under heavy loads or high speeds. In such conditions, the metal surfaces may come into direct contact, leading to rapid wear or even localized melting and welding of gear teeth.

Cleanliness is critical—**grit or metallic particles** in the lubricant act as abrasives, accelerating tooth surface damage. Therefore, gear lubricants must be filtered and kept free from contaminants. Additionally, the lubricant also plays a crucial role in **heat dissipation**, carrying away the thermal energy generated during meshing.

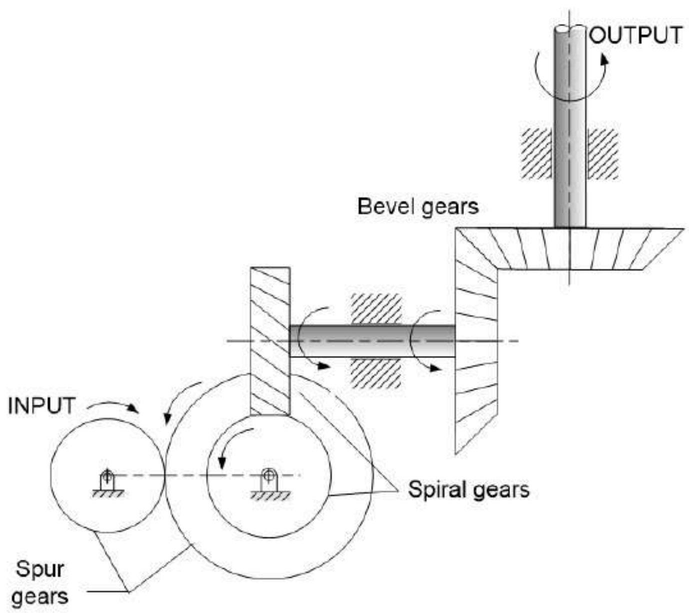
In practice, **thicker oils** such as SAE 30 are used in high-load applications, and gear capacity before scoring is affected by many variables including temperature, surface finish, and lubricant chemistry. For instance, oil films tend to become thinner at the pitch surface where rolling is predominant and thicker at the tips of the teeth where sliding action is greatest. As operating temperatures rise (e.g., beyond 350°F), the load-carrying ability of many lubricants increases due to the formation of protective carbonaceous layers.

Interestingly, **tooth load capacity increases** with speed up to a certain limit—about 4000 feet per minute (fpm) in tests—and peaks around 15,000 fpm before decreasing at higher speeds due to limitations in oil film formation. At extremely high speeds (e.g., over 30,000 fpm), specialized lubricants and gear designs are necessary to ensure reliability.

Finally, the design of the gear case must support effective lubrication by maintaining proper **gear alignment** and **rigidity**. Misalignment can cause uneven film distribution and gear wear. In conclusion, with the correct lubricant, careful maintenance, and precise alignment, a well-designed gear system can operate for an **indefinite life** with minimal wear.

GEAR TRAINS

Gear trains are combinations of two or more gears which are arranged to transmit power from the input shaft to the output shaft. The gears in a gear train can be a combination of different types of gears.

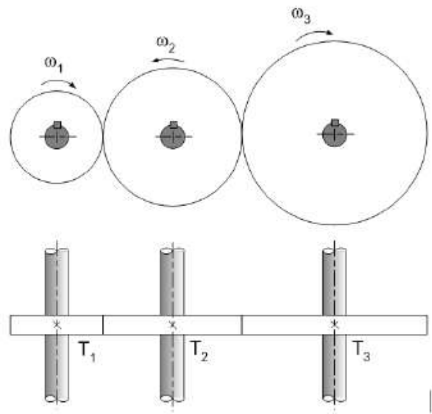


Gear trains are used when there is a need to orient the input and output shafts for specific spatial requirements and when there is a wide difference in speed ratio between the input and output shafts. In gear trains, we assume, unless otherwise specified, that the modules of the gears are the same.

Common Types of Gear Trains

1. Simple gear train

In a simple gear train, there is only one gear per shaft.



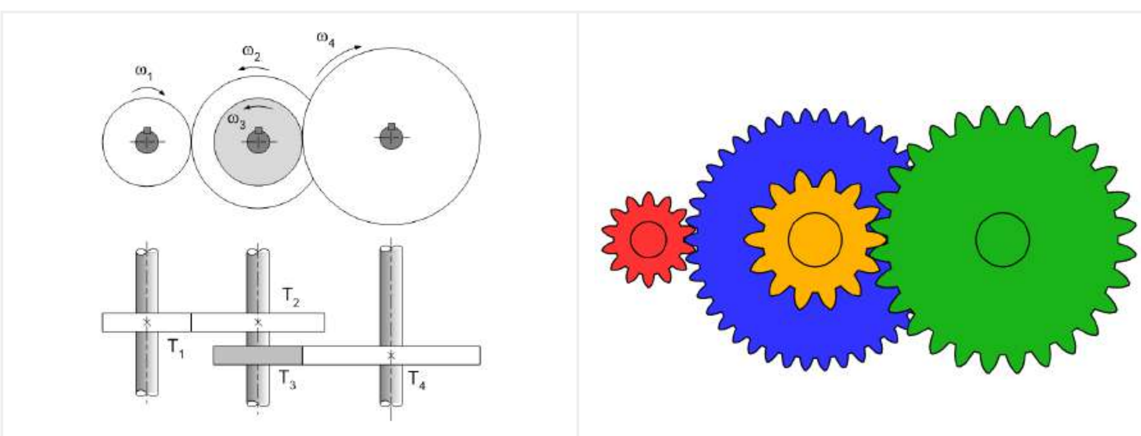
$$VR = \frac{\omega_1}{\omega_3} = \frac{T_3}{T_1}$$

$$VR = \frac{Speed\ of\ Driver\ Gear}{Speed\ of\ Driven\ Gear} = \frac{Teeth\ of\ Driven\ Gear}{Teeth\ of\ Driver\ Gear}$$

We can see that the velocity ratio of between the input and the output shaft of a simple gear train is independent of the intermediate gear 2, the intermediate gear will just change the direction of rotation of gear 3. The positive sign of the velocity ratio signifies that both the input gear 1 and output gear 2 are rotating in the same direction.

2. Compound gear train

In a compound gear train, there can be more than one gear per shaft.



$$VR = \frac{\omega_1}{\omega_4} = \frac{T_2 T_4}{T_1 T_3}$$

$$VR = \frac{Speed\ of\ Driver\ Gear}{Speed\ of\ Driven\ Gear} = \frac{Product\ of\ Teeth\ on\ Driven\ Gears}{Product\ of\ Teeth\ on\ Driver\ Gears}$$

MACHINE TOOL GEAR TRAIN APPLICATIONS

A gearbox is an enclosed metal casing of gear assembly that is used to generate several output speeds from one input speed, such as in automobile transmissions and machine tools.

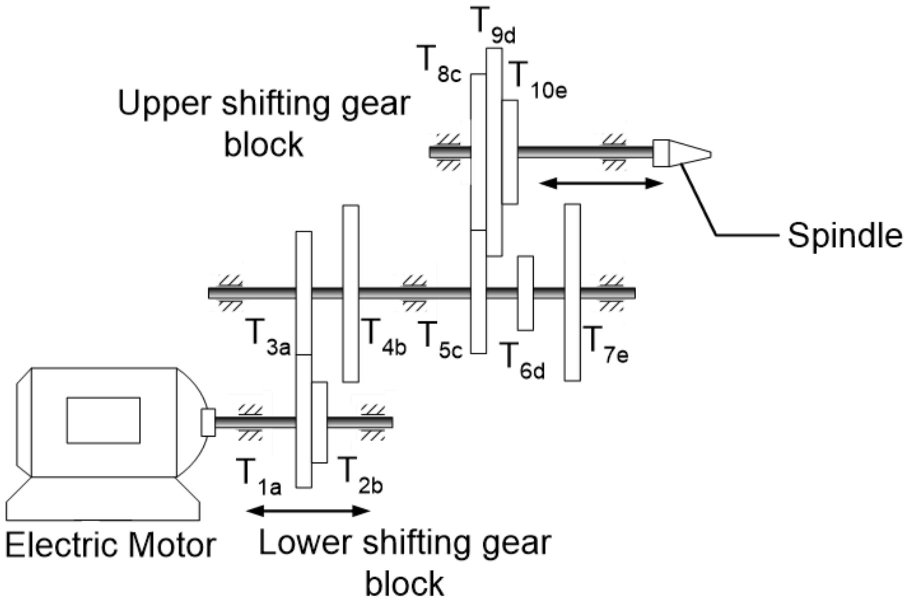
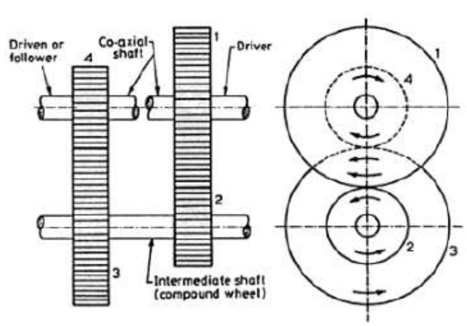


Figure above shows a layout of machine tool spindle with six speeds. In a machine tool gear box, we use a slide cluster to change the engagement of the gears. This is different with automobile gear boxes because in a machine tool gear box, we have to stop the machine first before we can change the gear engagement.

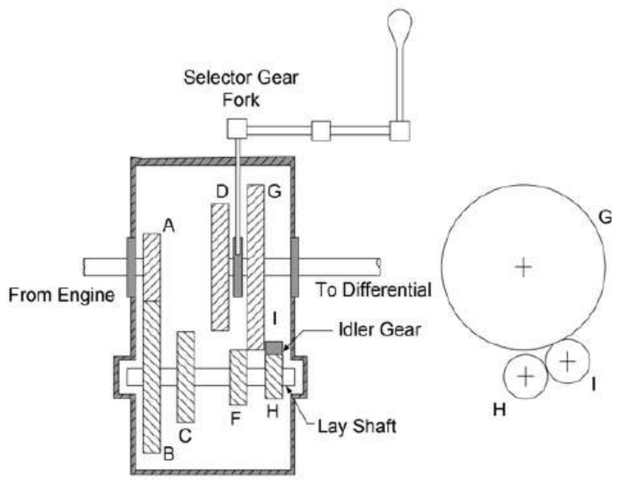
3. Reverted Gear Train

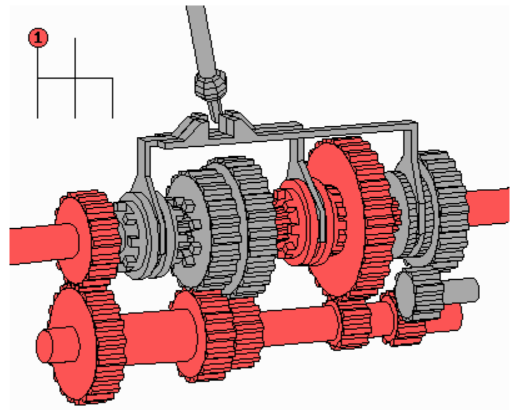
Reverted gear trains are used in the automobile transmission, lathe back gear, industrial speed reducers, and in clocks where the minute and hand hour hand shafts are coaxial.



A reverted gear train is a compound gear train in which the first and last gears are coaxial. Any Compound gear trains in which first and last gears are not co-axial, are called *non-reverted type gear trains*.

AUTOMOTIVE GEAR TRAIN APPLICATIONS

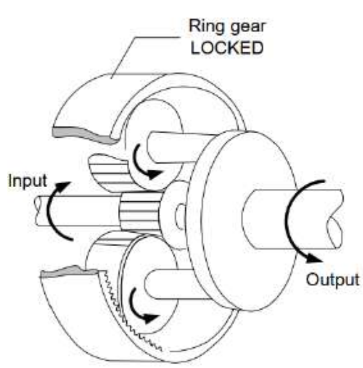


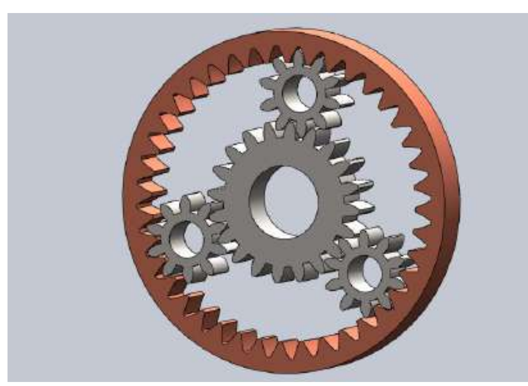


A lay shaft, also known as a countershaft, is an intermediate shaft used in multi-speed gearboxes, such as those found in automotive transmissions. It is responsible for transferring power from the input shaft (connected to the engine) to the output shaft (connected to the differential or wheels) through various gear combinations.

4. Epicyclic Gear Train

The key difference between a manual and an automatic transmission is that the manual transmission locks and unlocks different sets of gears to the output shaft to achieve the various gear ratios, while in an automatic transmission, the same set of gears produces all of the different gear ratios.
The epicyclic gear train or also known as planetary gearset is the device that makes this possible in an automatic transmission.

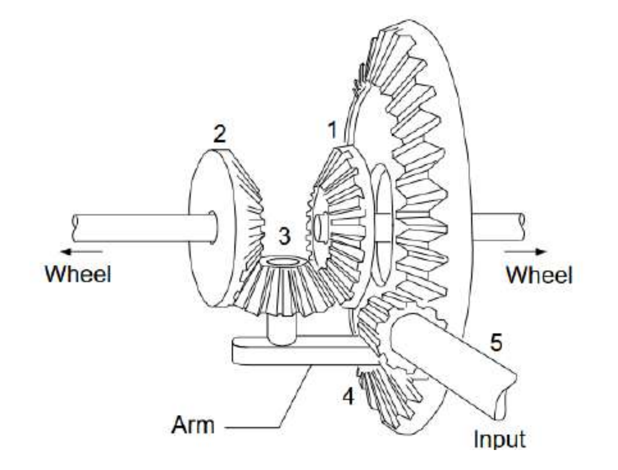


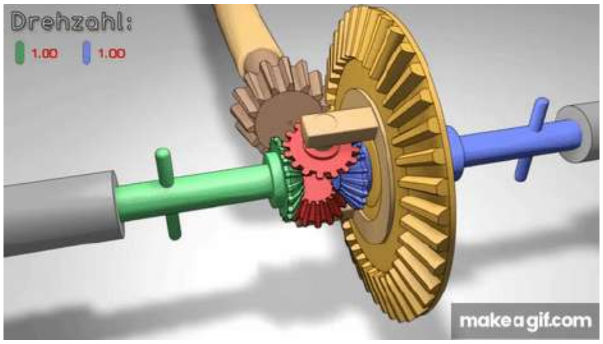


The number of degrees of freedom of a mechanism refers to the number of independent motions it can have. In the case of an epicyclic gear train, the number of degrees of freedom is 2.

5. Differential Gear Train

A differential gear train is an epicyclic gear train that allows the shafts to rotate at different speeds. Differential gear trains are found in automobiles.





OTHER TYPES OF AUTOMOTIVE TRANSMISSION

Auto Gear Shift (AGS) / Automated Manual Transmission (AMT)

Auto Gear Shift (AGS), also known as Automated Manual Transmission (AMT), is a type of transmission that automates the operation of a conventional manual gearbox. It retains the same gear and clutch arrangement as a manual transmission but replaces the need for driver input with an electro-hydraulic actuator system. These actuators, controlled by an Electronic Control Unit (ECU), automatically engage and disengage the clutch and shift gears based on driving conditions, throttle input, and vehicle speed. From the driver's perspective, AGS provides the convenience of automatic driving, as there's no need to manually operate the clutch or change gears. Most systems also include a manual mode, where the driver can override the ECU and shift up or down using a simple toggle or gear lever.

AGS is especially popular in entry-level and budget vehicles due to its low cost, compact design, and fuel efficiency. It delivers the basic benefits of an automatic transmission without the added weight and expense of torque converters or CVTs. However, gear changes in AGS are typically slower and can feel jerky, especially during rapid acceleration or uphill driving. Despite these limitations, AGS is a practical solution for drivers who want automatic convenience in traffic but at a more affordable price point. It strikes a balance between manual control and automated ease, making it well-suited for city driving in emerging markets and cost-sensitive segments.

Continuously Variable Transmission (CVT)

A **Continuously Variable Transmission (CVT)** is an automatic transmission system that offers a seamless and stepless range of gear ratios, unlike conventional transmissions that use a fixed number of gears. Instead of traditional gear sets, a CVT uses a system of two variable-diameter pulleys connected by a metal or high-strength rubber belt. One pulley is connected to the engine (the drive pulley), and the other is connected to the wheels (the driven pulley). As the vehicle accelerates or decelerates, hydraulic or electronic actuators adjust the width of the pulleys, causing the belt to ride higher or lower on each pulley, thereby continuously changing the gear ratio without discrete steps. This design enables the engine to operate at its most efficient RPM for a wide range of vehicle speeds, resulting in improved fuel efficiency and smoother acceleration.

CVTs are commonly found in small cars, hybrid vehicles, and scooters due to their simplicity and fuel-saving characteristics. Drivers often notice the absence of gear "shifts," which provides a smooth and uninterrupted driving experience. However, some may find the driving sensation unusual, especially during hard acceleration, as the engine sound may not correspond to increasing speed — a phenomenon sometimes referred to as the "rubber band effect." While CVTs are efficient and lightweight, they are generally not suitable for high-torque applications and may experience long-term reliability issues related to belt wear or overheating under heavy load. Nonetheless, for urban driving and fuel economy, CVT remains a practical and increasingly popular choice.