Part C: ENGINEERING ECONOMY

Fundamentals of Depreciation

In engineering economics, **depreciation** is defined as the systematic decrease in the value of a physical property or asset over time due to operational wear and tear, usage, and the simple passage of time. Conversely, **appreciation** represents the structural opposite of depreciation, denoting an increase in asset value over time. From a corporate and managerial perspective, tracking depreciation serves two critical purposes: first, it allows an enterprise to recover the initial capital invested in physical assets; second, it provides a mathematical basis to charge the cost of this value reduction directly to the price of the products or services produced by the asset.

To effectively implement depreciation models, a standard set of requirements dictates that a chosen method must be simple to calculate, must reliably recover the asset's capital, must be legally approved by regulatory bodies such as the Bureau of Internal Revenue (BIR), and must yield a book value that remains reasonably close to the actual market value of the property.

Standard Nomenclature and Terminologies

The mathematical formulations used to model asset degradation rely on a standardized set of variables:

- C_o: The original cost or first cost required to acquire and put the property into service.
- C_n: The book value or appraisal value of the property at the end of the n^{th} year.
- C_L: The salvage value, representing the residual value of the property at the end of its useful life.
- L: The total expected useful life of the property in years.
- d: The uniform annual depreciation charge (applicable in constant-rate models).
- d_n: The specific depreciation charge assessed during the n^{th} individual year.
- D_n: The total accumulated depreciation recorded from Year 1 up to the n^{th} year.
- D_L: The total accumulated depreciation recorded throughout the entire useful life up to the L^{th} year.
- i: The annual interest rate or time-value of money factor.
- k: The depreciation ratio or constant percentage rate applied to the beginning-of-year book value.
- Q_n: The number of physical units produced by the asset during the n^{th} year.
- T: The total number of units produced throughout the entire useful life of the property.
- H_n: The number of operational hours the asset is in service during the n^{th} year.
- H_T: The total number of service hours accumulated throughout the entire life of the property.

Comprehensive Analysis of Depreciation Methods

1. Straight Line Method (SLM)

The **Straight Line Method (SLM)** stands as the most extensively used method of depreciation across engineering industries primarily due to its mathematical simplicity. The foundational assumption of SLM is that the asset suffers a uniform annual depreciation charge throughout its entire operational lifecycle. This means the value drops by an identical monetary amount every single year.

To evaluate the asset at any intermediate n^{th} year, the book value (C_n) is found by subtracting the accumulated depreciation (D_n) from the first cost (C_o):

$$C_n = C_o - D_n$$

Because the value drops uniformly, the annual depreciation charge (d) can be computed by evaluating the total depreciable capital over the total life (L) or over an intermediate period (n):

$$d = \frac{C_o - C_L}{L} = \frac{C_o - C_n}{n}$$

At the end of the asset's useful life (L^{th} year), the final salvage value (C_L) is reached when the total lifetime accumulated depreciation (D_L) is subtracted from the original cost:

$$C_L = C_o - D_L$$

2. Sinking Fund Method (SFM)

The **Sinking Fund Method (SFM)** introduces the time value of money into asset degradation. It assumes that a uniform annual depreciation amount (d) is physically or conceptually deposited into an interest-bearing sinking fund. The interest earned on this fund compounds annually, meaning that the actual value reduction of the asset accelerates over time as the fund grows. The goal is to ensure that the accumulated value of the fund matches the total depreciation of the asset up to the period being considered.

When analyzing the asset at the intermediate n^{th} year, the total accumulated depreciation is equal to the future worth of an ordinary annuity. The difference between the original cost and the current book value (C_o - C_n) represents this accumulated fund value:

$$C_o - C_n = \frac{d}{i}\left[(1 + i)^n - 1\right]$$

Solving for the required periodic annual deposit (d) based on an intermediate book value gives:

$$d = \left(C_o - C_n\right) \frac{i}{(1+i)^n - 1}$$

If we look at the full operational horizon (L^{th} year), the total accumulated fund must equal the complete loss of value from the first cost to the salvage value (C_o - C_L):

$$C_o - C_L = \frac{d}{i}\left[(1 + i)^L - 1\right]$$

Consequently, the standard annual depreciation deposit required to fully recover the capital over the asset's complete lifecycle is formulated as:

$$d = \left(C_o - C_L\right)\frac{i}{(1+i)^L - 1}$$

3. Declining Balance Method (DBM)

The **Declining Balance Method (DBM)**, historically referred to as the constant percentage method, the fixed percentage method, or the Matheson formula method, is an accelerated depreciation model. It operates under the principle that the annual depreciation charge during any given year is a fixed, constant percentage (k) of the book value at the *beginning* of that specific year. Because the book value drops every year, the absolute monetary value of the annual depreciation charge (d_n) is highest in Year 1 and decreases progressively each subsequent year.

The specific depreciation charge (d_n) during the n^{th} year is computed by applying the rate k to the remaining book value from the previous year (C_{n-1}):

$$d_n = k\,C_o(1 + k)^{n-1}$$

Similarly, at the final year of operations (L^{th} year), the depreciation charge (d_L) is expressed as:

$$d_L = k\,C_o(1 + k)^{L-1}$$

To determine the constant depreciation rate (k) required to transition from the first cost to a specific intermediate book value or the final salvage value, the relationships are isolated via roots:

$$k = 1 - \left(\frac{C_n}{C_o}\right)^{\frac{1}{n}} = 1 - \left(\frac{C_L}{C_o}\right)^{\frac{1}{L}}$$

The comprehensive mathematical relationship governing the intermediate book value (C_n) relative to the final salvage value (C_L) across the asset's timeline is modeled as:

$$C_n = C_o\left(\frac{C_L}{C_o}\right)^{\frac{n}{L}}$$

Limitation: A fundamental constraint of the Declining Balance Method is that if an asset has a salvage value of zero (C_L = 0), the ratio $\frac{C_L}{C_o}$ becomes zero, rendering the calculation of the rate k mathematically impossible (k = 1 - 0 = 1, meaning the asset would fully depreciate to 0 in the very first year). Therefore, DBM cannot be utilized for capital recovery if the property possesses no salvage value.

4. Double Declining Balance Method (DDBM)

The **Double Declining Balance Method (DDBM)** is an aggressive variant of the standard declining balance system. It follows the exact same geometric progression mechanics as DBM, with the explicit operational modification that the depreciation rate (k) is predefined as exactly double the straight-line rate. Thus, the constant ratio is substituted directly as:

$$k = \frac{2}{L}$$

By replacing k with $\frac{2}{L}$, the formulas for intermediate book value (C_n) and the specific depreciation charge (d_n) during the n^{th} year become:

$$C_n = C_o\left(1 - \frac{2}{L}\right)^n$$

$$d_n = \frac{2}{L}C_o\left(1 - \frac{2}{L}\right)^{n-1}$$

Like other accelerated methods, the classic accounting identity remains intact: the book value at any point is the first cost minus the total accumulated depreciation ($C_n = C_o - D_n$), where the total accumulated loss is the simple sum of each preceding year's individual charges:

$$D_n = d_1 + d_2 + d_3 + \ldots + d_n$$

5. Sum-of-the-Years-Digit Method (SYDM)

The **Sum-of-the-Years-Digit Method (SYDM)** is another highly structured accelerated depreciation technique. Under SYDM, the annual depreciation charge is calculated by multiplying the total depreciable capital (C_o - C_L) by a changing fractional factor. This fraction is composed of a fixed denominator, which is the sum of the digits of the total years of useful life, and a variable numerator, which represents the remaining useful life digits counted in reverse order.

The individual depreciation charge (d_n) for any given n^{th} year is expressed as:

$$d_n = \frac{reversed\ digit}{sum\ of\ digits}\left(C_o - C_L\right)$$

Where the denominator represents the arithmetic series sum of the total lifespan:

$$sum\ of\ digits = 1 + 2 + 3 + ... + L = \frac{L(L+1)}{2}$$

And the numerator represents the remaining lifespan at the start of that period:

$$reversed\ digit = L - n + 1$$

To determine the intermediate status of the asset, the accumulated depreciation (D_n) and the resulting book value (C_n) are tracked through the summation of these declining annual increments:

$$D_n = d_1 + d_2 + d_3 + . . . + d_n$$

$$C_n = C_o - D_n$$

6. Service Output Method (SOM)

The **Service Output Method (SOM)** abandons time-based assumptions entirely, recognizing that an asset's degradation is often directly driven by physical workload rather than the mere calendar timeline. SOM assumes that the asset's depreciation charge during a given period is directly proportional to the number of industrial units produced or the total number of operational hours the asset was actively in service.

When using the total number of physical **production units** as the basis, the depreciation charge (d_n) for the n^{th} year relies on the ratio of the year's output (Q_n) to the total expected lifetime output (T):

$$d_n = \frac{Q_n}{T}\left(C_o - C_L\right)$$

Where the total expected structural production lifetime output (T) is the sum of all individual annual outputs across the asset's life:

$$T = \sum_{1}^{L} Q_L = Q_1 + Q_2 + Q_3 + \ ... \ + Q_L$$

If the operating timeline is instead tracked using active **service hours**, the formula substitutes the year's operational hours (H_n) against the total expected lifetime service hours (H_T):

$$d_n = \frac{H_n}{H_T}\left(C_o - C_L\right)$$

Consistent with standard asset tracking, the intermediate financial standing of the asset is verified by summing the production-driven losses to isolate the remaining book value:

$$D_n = d_1 + d_2 + d_3 + ... + d_n$$

$$C_n = C_o - D_n$$

Capitalized Cost and Capital Recovery

Problem Illustration: An engineering enterprise plans to install a specialized industrial pump system. The initial purchase and installation cost is **₱150,000**. The system has an estimated useful life of **6 years**, after which it will be sold for a salvage value of **₱30,000**. Annual operation and maintenance expenses are projected to be **₱12,000** paid at the end of every year indefinitely. With an interest rate of **10% compounded annually**, determine:

1. The **Capitalized Cost (CC)** of the system for perpetual operations.
2. The **Capital Recovery (CR)** using the **Sinking Fund Method**.
3. The **Capital Recovery (CR)** using the **Straight Line Method**.

Part 1: Capitalized Cost (CC) represents the sum of the first cost, the present worth of perpetual operation and maintenance expenses, and the present worth of perpetual replacement expenses. It consolidates the initial capital outlays and all future recurring expenditures into a single equivalent lump sum at Year 0, effectively representing the large endowment fund required today to establish, run, and periodically rebuild the system indefinitely.

$$CC = C_O + P_{OM} + \frac{S}{(1+i)^L - 1}$$

Where:

- C_o = ₱150,000 (the initial investment or first cost)
- $P_{OM} = \frac{OM}{i}$ (present worth of perpetual operation and maintenance)
- $S = C_o - C_L$ (the net amount needed to replace the property at the end of its life)
  To fund this replacement cycle indefinitely every L years, a capital sum is divided by the compound interest accumulator factor $(1 + i)^L - 1$, which yields the present worth of an infinite series of replacement costs.
- i = 0.10 (interest rate per year)
- L = 6 years (the operational lifecycle of the asset)

First, we find the net replacement cost (S), which represents the actual capital gap that must be filled every L years after utilizing the salvage value (C_L) from the old asset:

$$S = C_o - C_L = ₱150{,}000 - ₱30{,}000 = ₱120{,}000$$

Second, we calculate the present worth of perpetual operation and maintenance (POM) using the perpetuity concept:

$$P_{OM} = \frac{OM}{i} = \frac{₱12{,}000}{0.10} = ₱120{,}000$$

Now, we substitute these components into the primary capitalized cost formula to determine the infinite endowment required:

$$CC = ₱150,000 + ₱120,000 + \frac{₱120{,}000}{(1+0.10)^{6} - 1}$$

$$CC = ₱425,528.85$$

This is the **total lifetime cost** of owning, running, and replacing this pump system **forever**, compressed into today's money (Year 0).

Imagine you are setting up a permanent trust fund for this project. If you deposit ₱425,528.85 into a bank account that earns 10% annual interest today, that fund can buy the first pump for ₱150,000, pay the ₱12,000 maintenance fee every single year forever, and have exactly enough money accumulated to buy a brand-new ₱120,000 replacement pump every 6 years without ever running out of money. It represents the true total financial commitment of a perpetual project.

Part 2: Capital Recovery (CR)

While Capitalized Cost translates future infinite liabilities into a present lump sum, **Capital Recovery (CR)** executes the reverse process by transforming asset costs into an equivalent annual series. Capital recovery is the standard term used to describe the annual economic cost of recovering the invested capital over the lifetime of the asset. It represents the minimum uniform annual revenue or savings an asset must generate to justify its initial expenditure and cover its loss of value over time. Depending on the accounting framework and interest treatments applied, capital recovery can be calculated using two primary methodologies: the Sinking Fund Method and the Straight Line Method.

CR: Sinking Fund Method

The **Sinking Fund Method** incorporates compound interest directly into the annual depreciation model. Under this approach, the Capital Recovery cost is defined as the mathematical sum of the annual depreciation annuity plus the annual interest earned on the initial capital investment:

$$CR = Annual\ Depreciation + Interest\ on\ Capital$$

By substituting the sinking fund depreciation formula for the annual depreciation term and multiplying the original cost by the interest rate, the comprehensive formula is established as:

$$CR = \frac{S \cdot i}{(1+i)^{L} - 1} + \left(C_o i\right)$$

Using the established parameters (S = ₱120,000, C_o = ₱150,000, i = 0.10, L = 6):

$$CR = \frac{(₱120{,}000)\ (0.10)}{(1+0.10)^{6} - 1} + (₱150,000)(0.10)$$

CR = ₱30,552.89 per year

To legally and financially justify owning this pump, the machinery needs to generate at least **₱30,552.89 in net revenue or savings every single year**. This annual amount covers two things: it pays the 10% interest on the ₱150,000 tied up in the machinery, and it sets aside a portion into a "sinking fund" so that when the pump dies in Year 6, you have exactly enough cash to replace it.

CR: Straight Line Method

The **Straight Line Method** offers an alternative accounting approach to capital recovery. Instead of compounding a sinking fund, it estimates the annual cost of asset ownership as the uniform straight-line depreciation plus the *average interest* earned over the lifespan. The formula provided in your review material is:

$$CR = Annual\ Depreciation + Average$$

The mathematical expression provided for this specific method is formulated as:

$$CR = \frac{S \cdot i}{2}\left(\frac{L+1}{L}\right) + \left(C_o i\right)$$

This equation focuses on computing the average interest cost associated with the asset's declining book value. By multiplying the average depreciable capital by the interest factor and adjusting for the time intervals via the $\left(\frac{L+1}{L}\right)$ bracket, it approximates the annual interest burden alongside the salvage value's individual interest yield.

Substituting our problem values into this linear approximation model yields:

$$CR = \frac{(₱120{,}000)\ (0.10)}{2}\left(\frac{6+1}{6}\right) + (₱150,000)(0.10)$$

CR = ₱10,000.00 per year

This represents a simplified, linear approximation of the **average annual interest cost** associated with the asset's declining value. Unlike the sinking fund method which assumes you are compounding interest in a bank, this formula calculates the average interest tied up in the asset over its lifespan. It tells management that, on average, ₱10,000 of interest is lost or paid per year purely due to capital being trapped inside a depreciating asset.

Capital Financing and Types of Capital

In engineering economy and project management, establishing a commercial enterprise or launching a large-scale engineering project requires the strategic mobilization of financial resources, a process known as **Capital Financing**. To fund operations, procurement, and infrastructure development, firms rely on **two fundamental types of capital**: **borrowed capital** and **equity capital**.

- **Borrowed Capital:** This refers to funds obtained from external lending sources, such as commercial banks, institutional investors, or through issuing debt instruments. Borrowed capital creates a legal obligation where the firm must pay back the principal along with a specified interest rate over a fixed period, regardless of whether the business turns a profit.
- **Equity Capital:** This represents capital supplied by the owners, partners, or stockholders of the business. Unlike debt, equity capital does not require compulsory periodic interest payments; instead, investors receive a share of the ownership and returns through corporate dividends or capital appreciation.

The Three Types of Business Organizations

The manner in which capital is raised, legal liabilities are allocated, and corporate decisions are executed depends directly on how the enterprise is legally structured. In commercial practice, there are **three primary types of business organizations**: **sole proprietorship**, **partnership**, and **corporation**.

- **Sole Proprietorship:** This is a basic form of business organization characterized by having **only one individual owner**. In a sole proprietorship, the owner possesses total managerial control, receives all corporate profits, and assumes unlimited personal liability for all debts, losses, and legal obligations incurred by the business enterprise.
- **Partnership:** A partnership is an association **owned by two or more persons** who pool their capital, skills, and resources to co-own a business. The partners operate under a shared agreement regarding profit distribution and management, though they generally face shared personal liability for the financial and legal obligations of the firm.
- **Corporation:** A corporation is a highly structured form of business organization that functions as a **separate legal entity, entirely distinct from its individual owners** (the shareholders). Because the corporation is recognized legally as an independent "person," it can own property, enter into contracts, incur debt, and be held legally liable in court. This distinct legal separation grants shareholders the crucial advantage of limited liability, meaning their personal assets are protected if the corporation goes bankrupt.

Fundamentals of Bonds

When a corporation or a government entity needs to secure massive amounts of long-term borrowed capital to fund major infrastructure or industrial expansions, it often issues a debt security known as a **bond**. By definition, a bond is a formal **certificate of indebtedness** issued by the borrower to the investor, typically structured to **mature over a long-term horizon, usually in not less than 10 years**.

When an investor purchases a bond, they are lending money to the issuer. In return, the issuer legally agrees to pay the investor periodic interest payments (known as coupon payments) throughout the life of the bond, and then return a lump-sum amount (the redemption value) when the bond reaches its final maturity date.

Bond Value (P)

From an engineering economy standpoint, determining how much an investor should pay for a bond today requires calculating its **Bond Value (P)**. The bond value is defined as the **present worth of the entire expected amount to be received through the ownership of a bond**. This means a bond's current worth is found by discounting its future cash flows—specifically, the stream of periodic coupon interest payments and the final lump-sum redemption payment—back to Year 0 based on the investor's required market interest rate.

The cash flow behaves as a combination of two elements: the periodic interest payments form a standard ordinary annuity, while the final redemption price behaves as a single future payment. By combining the ordinary annuity formula with the single-payment present worth formula, we establish the standard governing bond valuation equation:

$$P = \frac{Fr}{i}\left[1 + (1 + i)^{-N}\right] + C(1 + i)^{-N}$$

Where the structural variables are defined as:

- P: The **bond value**, representing the fair purchase price or present worth today.
- F: The **face value or par value** printed on the bond certificate, which serves as the base for calculating periodic interest.
- r: The **bond rate per period** (coupon rate), expressed as a percentage of the face value. Therefore, the actual dollar or peso amount of each periodic payment received by the investor is calculated as $F \cdot r$.
- i: The **interest rate per period** representing the investor's required yield, market interest rate, or target rate of return.
- N: The total **number of periods** or interest payment intervals remaining until the bond reaches maturity.
- C: The **redemption value**, which is the final lump-sum payment paid back to the bondholder at the maturity date.

In many standard exam problems, a bond is described as being "redeemed at par." When this phrase is used, it means the final redemption value is exactly equal to the initial face value (C = F). If the bond is redeemed at a premium or discount, the problem will explicitly state a value for C that differs from F. Always verify whether the time intervals (e.g., annual vs. semi-annual payments) require you to divide the annual interest rates (r and i) and multiply the number of years to find the correct per-period values for N.

Inflation and Purchasing Power

In the engineering economy, financial evaluations cannot be conducted in a static environment where the purchasing power of money remains completely fixed. Projects spanning multiple years or decades must account for **inflation**. By definition, inflation is the decrease in the value of currency due to the increase in the value of currency circulated. This structural expansion of money in circulation reduces its purchasing power, which will eventually cause the increase in the prices of commodities. For an engineer designing long-term systems, developing municipal infrastructure, or drafting cost estimates, ignoring this macroeconomic force leads to an underestimation of future operational expenditures and a miscalculation of project feasibility.

The Mathematical Model for Price Escalation

To model the impact of inflation on individual goods, services, or raw materials over a multi-year horizon, engineering economics treats the price escalation identically to compound interest. As the value of currency drops, the price of a specific commodity rises exponentially according to a constant annual rate. The governing mathematical formula used to predict the future cost of goods under inflationary pressures is expressed as:

$$F = P(1 + i_f)^n$$

Where the structural variables are defined as follows:

- P: The current price of the commodity at the present time (t = 0).
- F: The price of the same commodity in the future after a given time horizon has elapsed.
- i_f: The constant inflation rate per year, representing the annual percentage increase in price levels.
- n: The number of years over which the inflationary price escalation takes place.

Real vs. Market Rates (Board Exam Integration)

When evaluating real-world cash flows under inflation, examiners frequently test an engineer's ability to distinguish between **actual (current) Peso** and **real (constant) Peso**.

- **Actual/Market Dollars:** This represents the actual number of currency units spent or received at a future date. It accounts for the inflated price tag (F) you will physically pay at the cash register in Year n.
- **Real/Constant Dollars:** This represents the purchasing power of that future money expressed in terms of baseline present-day money (Year 0 purchasing capacity).

Selection of Alternatives

In engineering economics, one of the primary responsibilities of an engineer is to evaluate competing engineering designs, systems, or projects and select the alternative that offers the maximum economic benefit or the lowest relative cost. The selection of alternatives can be made using specific quantitative criteria that compare financial returns, the speed of capital recovery, or total annual cost burdens. The attached review material establishes three formal criteria for making these decisions: **Rate of Return**, **Payout Period**, and **Annual Cost**.

1. Rate of Return Criterion

The **Rate of Return** method evaluates alternative investments based on their engineering efficiency in generating earnings relative to the size of the initial capital deployed. This metric is expressed as a percentage or decimal ratio and represents the profitability of an alternative. When choosing among multiple projects, management typically establishes a Minimum Attractive Rate of Return (MARR). An alternative is considered economically viable only if its calculated rate of return meets or exceeds this target baseline.

The governing formula for this calculation is structured as:

$$Rate\ of\ Return\ =\frac{Net\ Profit}{Total\ Investment}$$

Under this framework, the **Total investment** constitutes the complete capital outlay required to put the project into operation. The **Net profit** represents the annual net earnings derived after deducting all operational costs, depreciation, taxes, and secondary expenditures from gross revenues. When evaluating mutually exclusive alternatives, a higher rate of return indicates a more efficient utilization of corporate capital.

2. Payout Period Criterion

The **Payout Period** (often referred to as the payback period) measures the speed with which an alternative can recover its initial capital investment from its operational cash inflows. This criterion is particularly useful in industries where technology changes rapidly or in high-risk environments where a long capital lock-up is undesirable. It defines the number of years required for the net cash flows generated by the asset to equal the initial net investment outlays.

The formula provided to determine the payout timeline is:

$$Payout\ period\ =\frac{Total\ investment-Salvage\ value}{Net\ annual\ cash\ flow}$$

In this equation, the numerator represents the net unrecoverable capital that must be completely paid off through operations. By subtracting the **Salvage value** (the residual value recovered at the end of the asset's life) from the **Total investment**, we isolate the exact capital deficit that must be recovered. This net value is divided by the **Net annual cash flow**, which is the annual cash surplus generated by the project (typically computed as revenues minus operating expenses, before non-cash charges like depreciation are subtracted). When choosing between options under this criterion, the alternative with the **shortest payout period** is preferred because it minimizes financial risk and returns capital to the enterprise more rapidly.

3. Annual Cost Criterion

The **Annual Cost** method is an alternative selection tool, especially effective when evaluating options that have identical revenue streams or provide necessary services where revenue cannot be easily quantified (such as comparing two different ventilation systems or wastewater treatment setups). Instead of analyzing profits, this method converts all capital expenditures, interest obligations, and operational costs into an equivalent annual expenditures baseline. The alternative that yields the **lowest total annual cost** is selected as the most economically optimal choice.

According to the attached reference sheet, the comprehensive equation for determining this metric is:

$$Annual\ Cost = depreciation + interest\ on\ capital + O\&M + other\ out\ of\ the\ pocket\ expenses$$

To solve this formula, each cost layer must be accounted for:

- **Depreciation:** The annual cost attributed to the asset's decline in value over time, calculated using one of the primary methods established in Section 3 (such as the straight-line or sinking fund allocation).
- **Interest on Capital:** The annual opportunity cost or financing cost of the money tied up in the asset, determined by multiplying the investment value by the interest rate.
- **Operation and Maintenance (O&M):** The recurring annual cash expenses required to keep the physical asset functioning smoothly, including labor, power, and routine repairs.
- **Other Out-of-the-Pocket Expenses:** Any additional direct annual cash outlays associated with the asset, such as insurance premiums, property taxes, or administrative licensing fees.