In the study of material science and fluid mechanics, evaluating whether a structural element contains an internal defect, pocket, or hollow void relies heavily on the concepts of **bulk density** and **theoretical density**.

The Law of Density Consistency

Density (ρ) is an intrinsic physical property of a homogeneous material, meaning its value depends solely on the substance's chemical composition and molecular arrangement, completely independent of its shape or size. For pure aluminum, the standard theoretical density is given as ρ_{Al} = 2,700 kg/m^3.

Bulk vs. True Material Density

When analyzing a macro-geometry like a sphere, we distinguish between two volumetric states:

1. **Total Geometric Volume (V_{total}):** The physical space enclosed by the outer boundaries of the shape. For a perfect sphere, this is derived from the outer radius (r) via the equation:

   $$V_{total} = \frac{4}{3}\pi r^3$$

2. **True Material Volume (V_{mat}):** The volume actually occupied by solid aluminum molecules.

If a metallic object is solid and defect-free, its calculated bulk density $\left(\rho_{bulk} = \frac{m_{total}}{V_{total}}\right)$ must exactly match its intrinsic theoretical material density (ρ_{Al}). If $\rho_{bulk} < \rho_{Al}$, it establishes that the outer geometric shell encloses an internal void or a region filled with a negligible-mass fluid (like air). The missing mass (Δm) represents the amount of solid aluminum that *should* occupy that structural space but is absent.

An essential engineering shortcut is converting the density of water-referenced substances into CGS units directly. Since 1 g/cm^3 = 1,000 kg/m^3, the density of solid aluminum can be rewritten simply as:

$$\rho_{Al} = \frac{2{,}700\,\frac{kg}{m^3}}{1{,}000} = 2.70\frac{g}{cm^3}$$

<u>Step 1:</u> Calculate the Total Geometric Volume of the Sphere (V_{total})

Substitute the radius into the geometric formula for a sphere:

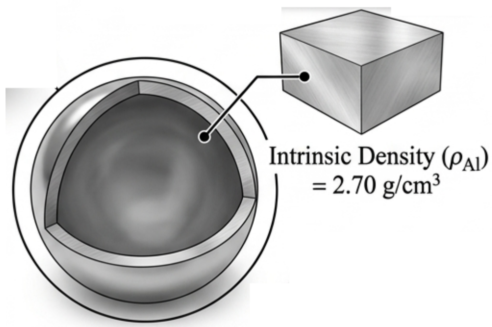
$$V_{total} = \frac{4}{3}\pi\left(0.125\ cm^3\right) = 0.5236\ cm^3$$

Step 2: **Determine if the Sphere is Solid or Hollow**

Calculate the bulk density of this specific sphere and compare it to the known density of solid aluminum:

$$\rho_{bulk} = \frac{m_{sphere}}{V_{total}} = \frac{0.35\ g}{0.5236\ cm^3} = 0.6685\frac{g}{cm^3}$$

Since $\rho_{bulk} < \rho_{Al}$, **the sphere is hollow**



Step 3: **Quantify the Missing Mass of Aluminum (Δm)**

First, determine the ideal total mass (m_{ideal}) that this sphere *would* have if it were completely solid:

$$m_{ideal} = \rho_{Al}V_{total} = \left(2.70\frac{g}{cm^3}\right)\left(0.5236\ cm^3\right) = 1.4137\ g$$

Now, subtract the actual measured mass of the hollow sphere to find the missing mass:

$$\Delta m = 1.4137\ g - 0.35g = 1.0637\ g$$